Quantum non-abelian localization: Conjectures and partial results

Chris Woodward,Rutgers

joint with E. Gonzalez

Question: how are equivariant Gromov-Witten invariants of a Hamiltonian G-manifold related to those of its symplectic quotient?

thanks to C. Teleman

Notation

G is a compact, 1-connected Lie group with Lie alg $\mathfrak{g}$

Notation

G is a compact, 1-connected Lie group with Lie alg $\mathfrak{g}$

T a maximal torus with Lie alg $\mathfrak{t}$

Notation

G is a compact, 1-connected Lie group with Lie alg $\mathfrak{g}$

T a maximal torus with Lie alg $\mathfrak{t}$

$W = N(T)/T$ its Weyl group

Notation

G is a compact, 1-connected Lie group with Lie alg $\mathfrak{g}$

T a maximal torus with Lie alg $\mathfrak{t}$

$W = N(T)/T$ its Weyl group

X a compact Hamiltonian G-manifold

Notation

G is a compact, 1-connected Lie group with Lie alg $\mathfrak{g}$

T a maximal torus with Lie alg $\mathfrak{t}$

$W = N(T)/T$ its Weyl group

X a compact Hamiltonian G-manifold

J invariant compatible almost complex str.

Notation

G is a compact, 1-connected Lie group with Lie alg $\mathfrak{g}$

T a maximal torus with Lie alg $\mathfrak{t}$

$W = N(T)/T$ its Weyl group

X a compact Hamiltonian G-manifold

J invariant compatible almost complex str.

$\Phi : X \to \mathfrak{g}^*$ moment map

Notation

G is a compact, 1-connected Lie group with Lie alg $\mathfrak{g}$

T a maximal torus with Lie alg $\mathfrak{t}$

$W = N(T)/T$ its Weyl group

X a compact Hamiltonian G-manifold

J invariant compatible almost complex str.

$\Phi : X \to \mathfrak{g}^*$ moment map

$X/\!\!/G = \Phi^{-1}(0)/G$ symplectic quotient

Equivariant quantum cohomology

$QH(X)$ counts holomorphic maps $u : \mathbb{P}^1 \longrightarrow X$

Equivariant quantum cohomology

$QH(X)$ counts holomorphic maps $u : \mathbb{P}^1 \longrightarrow X$

$QH_G(X)^{Givental}$ counts holomorphic maps $u : \mathbb{P}^1 \longrightarrow X$ equivariantly

$QH_G(X)$ *should* count holomorphic maps $u : \mathbb{P}^1 \longrightarrow X_{G_{\mathbb{C}}} := X \times_{G_{\mathbb{C}}} EG_{\mathbb{C}}$

Equivariant quantum cohomology

$QH(X)$ counts holomorphic maps $u : \mathbb{P}^1 \longrightarrow X$

$QH_G(X)^{Givental}$ counts holomorphic maps $u : \mathbb{P}^1 \longrightarrow X$ equivariantly

$QH_G(X)$ *should* count holomorphic maps
$u : \mathbb{P}^1 \longrightarrow X_{G_\mathbb{C}} := X \times_{G_\mathbb{C}} EG_\mathbb{C}$

i.e. maps $v : \mathbb{P}^1 \longrightarrow BG_\mathbb{C}$ plus lifts to $X_{G_\mathbb{C}}$

Equivariant quantum cohomology

$QH(X)$ counts holomorphic maps $u : \mathbb{P}^1 \longrightarrow X$

$QH_G(X)^{Givental}$ counts holomorphic maps $u : \mathbb{P}^1 \longrightarrow X$ equivariantly

$QH_G(X)$ *should* count holomorphic maps $u : \mathbb{P}^1 \longrightarrow X_{G_\mathbb{C}} := X \times_{G_\mathbb{C}} EG_\mathbb{C}$

i.e. maps $v : \mathbb{P}^1 \longrightarrow BG_\mathbb{C}$ plus lifts to $X_{G_\mathbb{C}}$

i.e. holomorphic $G_\mathbb{C}$-bundles $P \longrightarrow X$ with sections $u : \mathbb{P}^1 \longrightarrow P \times_{G_\mathbb{C}} X$

Equivariant quantum cohomology

$QH(X)$ counts holomorphic maps $u : \mathbb{P}^1 \longrightarrow X$

$QH_G(X)^{Givental}$ counts holomorphic maps $u : \mathbb{P}^1 \longrightarrow X$ equivariantly

$QH_G(X)$ *should* count holomorphic maps $u : \mathbb{P}^1 \longrightarrow X_{G_\mathbb{C}} := X \times_{G_\mathbb{C}} EG_\mathbb{C}$

i.e. maps $v : \mathbb{P}^1 \longrightarrow BG_\mathbb{C}$ plus lifts to $X_{G_\mathbb{C}}$

i.e. holomorphic $G_\mathbb{C}$-bundles $P \longrightarrow X$ with sections $u : \mathbb{P}^1 \longrightarrow P \times_{G_\mathbb{C}} X$

i.e. triples (P, A, u) of principal G-bundle P, connection A, and holomorphic $u : \mathbb{P}^1 \longrightarrow P \times_G X$

Gauged holomorphic maps

Algebraic approach still on X a point (moduli of bundles on a nodal curve? See e.g. Frenkel-Teleman to appear)

Gauged holomorphic maps

Algebraic approach still on X a point (moduli of bundles on a nodal curve? See e.g. Frenkel-Teleman to appear)

Symplectic approach: Fix P principal G-bundle

Gauged holomorphic maps

Algebraic approach still on X a point (moduli of bundles on a nodal curve? See e.g. Frenkel-Teleman to appear)

Symplectic approach: Fix P principal G-bundle

$\mathcal{A}(P, X) = \{(A, u), \overline{\partial}_A u = 0\}$ space of gauged hol. maps

Gauged holomorphic maps

Algebraic approach still on X a point (moduli of bundles on a nodal curve? See e.g. Frenkel-Teleman to appear)

Symplectic approach: Fix P principal G-bundle

$\mathcal{A}(P, X) = \{(A, u), \overline{\partial}_A u = 0\}$ space of gauged hol. maps

$G(P)$ group of gauge transformations acts on $\mathcal{A}(P, X)$

Gauged holomorphic maps

- Algebraic approach still on X a point (moduli of bundles on a nodal curve? See e.g. Frenkel-Teleman to appear)
- Symplectic approach: Fix P principal G-bundle
- $\mathcal{A}(P, X) = \{(A, u), \overline{\partial}_A u = 0\}$ space of gauged hol. maps
- $G(P)$ group of gauge transformations acts on $\mathcal{A}(P, X)$
- moment map $\mathcal{A}(P, X) \to \Omega^2(\mathbb{P}^1, P(\mathfrak{g})), (A, u) \mapsto F_A + \operatorname{Vol} u^* \Phi$

Gauged holomorphic maps

- Algebraic approach still on X a point (moduli of bundles on a nodal curve? See e.g. Frenkel-Teleman to appear)
- Symplectic approach: Fix P principal G-bundle
- $\mathcal{A}(P,X) = \{(A,u), \overline{\partial}_A u = 0\}$ space of gauged hol. maps
- $G(P)$ group of gauge transformations acts on $\mathcal{A}(P,X)$
- moment map $\mathcal{A}(P,X) \to \Omega^2(\mathbb{P}^1, P(\mathfrak{g})), (A,u) \mapsto F_A + \mathrm{Vol}\, u^*\Phi$
- Depends on two-form $\mathrm{Vol} \in \Omega^2(\mathbb{P}^1)$ and inner product $\mathfrak{g} \times \mathfrak{g} \to \mathbb{R}$.

Symplectic vortices

$$M(P,X) := \mathcal{A}(P,X)/\!\!/G(P) = \{F_A + \epsilon^{-1}\,\mathrm{Vol}\,u^*\Phi = 0\}/G(P)$$

Symplectic vortices

$M(P,X) := \mathcal{A}(P,X) /\!\!/ G(P) = \{F_A + \epsilon^{-1} \operatorname{Vol} u^*\Phi = 0\}/G(P)$

$M(P,X)$ is the *moduli space of symplectic vortices*

Symplectic vortices

$M(P,X) := \mathcal{A}(P,X)/\!\!/G(P) = \{F_A + \epsilon^{-1}\,\mathrm{Vol}\,u^*\Phi = 0\}/G(P)$

$M(P,X)$ is the *moduli space of symplectic vortices*

Studied by Salamon, Mundet, Cieliebak, Gaio, ...

Symplectic vortices

$M(P,X) := \mathcal{A}(P,X)/\!\!/G(P) = \{F_A + \epsilon^{-1}\,\mathrm{Vol}\,u^*\Phi = 0\}/G(P)$

$M(P,X)$ is the *moduli space of symplectic vortices*

Studied by Salamon, Mundet, Cieliebak, Gaio, ...

Compactification $\overline{M}(P,X)$ obtained by allowing sphere bubbles in fibers of $P(X)$

Symplectic vortices

$M(P,X) := \mathcal{A}(P,X)/\!\!/G(P) = \{F_A + \epsilon^{-1}\,\mathrm{Vol}\,u^*\Phi = 0\}/G(P)$

$M(P,X)$ is the *moduli space of symplectic vortices*

Studied by Salamon, Mundet, Cieliebak, Gaio, ...

Compactification $\overline{M}(P,X)$ obtained by allowing sphere bubbles in fibers of $P(X)$

Question: how does $\overline{M}(P,X)$ depend on ϵ?

Finite Dimensional Analogy

X_0, X_1 Hamiltonian G-manifolds

Finite Dimensional Analogy

X_0, X_1 Hamiltonian G-manifolds

$X = X_0 \times X_1$ with two form $\omega_0 + \epsilon^{-1}\omega_1$

Finite Dimensional Analogy

X_0, X_1 Hamiltonian G-manifolds

$X = X_0 \times X_1$ with two form $\omega_0 + \epsilon^{-1}\omega_1$

diagonal action of G has moment map $\Phi_0 + \epsilon^{-1}\Phi_1$

Finite Dimensional Analogy

X_0, X_1 Hamiltonian G-manifolds

$X = X_0 \times X_1$ with two form $\omega_0 + \epsilon^{-1}\omega_1$

diagonal action of G has moment map $\Phi_0 + \epsilon^{-1}\Phi_1$

$X /\!\!/ G$ is a X_0 bundle over $X_1 /\!\!/ G$ for ϵ small

Finite Dimensional Analogy

X_0, X_1 Hamiltonian G-manifolds

$X = X_0 \times X_1$ with two form $\omega_0 + \epsilon^{-1}\omega_1$

diagonal action of G has moment map $\Phi_0 + \epsilon^{-1}\Phi_1$

$X /\!\!/ G$ is a X_0 bundle over $X_1 /\!\!/ G$ for ϵ small

$X /\!\!/ G$ is a X_1 bundle over $X_0 /\!\!/ G$ for ϵ large

Finite Dimensional Analogy

X_0, X_1 Hamiltonian G-manifolds

$X = X_0 \times X_1$ with two form $\omega_0 + \epsilon^{-1}\omega_1$

diagonal action of G has moment map $\Phi_0 + \epsilon^{-1}\Phi_1$

$X /\!\!/ G$ is a X_0 bundle over $X_1 /\!\!/ G$ for ϵ small

$X /\!\!/ G$ is a X_1 bundle over $X_0 /\!\!/ G$ for ϵ large

In between have wall-crossing formulas, for example Kalkman

Infinite volume limit

studied by Gaio-Salamon

Infinite volume limit

studied by Gaio-Salamon

A 0-*vortex* is a collection of holomorphic maps to $X, X /\!\!/ G$, and vortices on $\mathbb{C}$.

Infinite volume limit

studied by Gaio-Salamon

A 0-*vortex* is a collection of holomorphic maps to $X, X/\!\!/G$, and vortices on $\mathbb{C}$.

$\overline{M}(P, X)_0$ moduli space of 0-vortices

Infinite volume limit

studied by Gaio-Salamon

A 0-*vortex* is a collection of holomorphic maps to $X, X/\!\!/G$, and vortices on $\mathbb{C}$.

$\overline{M}(P, X)_0$ moduli space of 0-vortices

Conjecture: $\overline{M}(P, X)_0$ compactifies $\overline{M}(P, X)_\epsilon$ as $\epsilon \to 0$

Infinite volume limit

studied by Gaio-Salamon

A 0-*vortex* is a collection of holomorphic maps to $X, X/\!\!/G$, and vortices on $\mathbb{C}$.

$\overline{M}(P, X)_0$ moduli space of 0-vortices

Conjecture: $\overline{M}(P, X)_0$ compactifies $\overline{M}(P, X)_\epsilon$ as $\epsilon \to 0$

Very similar results by Ziltener, who conjectures a quantum Kirwan map

$$Q\kappa_G : QH_G(X) \to QH(X/\!\!/G)$$

counting stable vortices on $\mathbb{C}$.

Infinite volume limit

studied by Gaio-Salamon

A 0-*vortex* is a collection of holomorphic maps to $X, X/\!\!/G$, and vortices on $\mathbb{C}$.

$\overline{M}(P, X)_0$ moduli space of 0-vortices

Conjecture: $\overline{M}(P, X)_0$ compactifies $\overline{M}(P, X)_\epsilon$ as $\epsilon \to 0$

Very similar results by Ziltener, who conjectures a quantum Kirwan map

$$Q\kappa_G : QH_G(X) \to QH(X/\!\!/G)$$

counting stable vortices on $\mathbb{C}$.

Zero volume limit

studied by Gonzalez-Woodward

Zero volume limit

studied by Gonzalez-Woodward

A ∞-*vortex* is a map $u : \mathbb{P}^1 \to X$ with $\int_{\mathbb{P}}^1 u^* \Phi \, \mathrm{Vol}_{\mathbb{P}^1} = 0$

Zero volume limit

studied by Gonzalez-Woodward

A ∞-*vortex* is a map $u : \mathbb{P}^1 \to X$ with $\int_{\mathbb{P}}^1 u^* \Phi \, \mathrm{Vol}_{\mathbb{P}^1} = 0$

Stable ∞-vortices allow sphere bubbling

Zero volume limit

studied by Gonzalez-Woodward

A ∞-*vortex* is a map $u : \mathbb{P}^1 \to X$ with $\int_{\mathbb{P}}^1 u^* \Phi \, \mathrm{Vol}_{\mathbb{P}^1} = 0$

Stable ∞-vortices allow sphere bubbling

$\overline{M}(X)_\infty$ moduli space of stable ∞-vortices

Zero volume limit

studied by Gonzalez-Woodward

A ∞-*vortex* is a map $u : \mathbb{P}^1 \to X$ with $\int_{\mathbb{P}}^1 u^* \Phi \, \mathrm{Vol}_{\mathbb{P}^1} = 0$

Stable ∞-vortices allow sphere bubbling

$\overline{M}(X)_\infty$ moduli space of stable ∞-vortices

G acts on $\overline{M}_{0,3}(X)$ and $\overline{M}(X)_\infty$ is the pre-symplectic quotient

$$\overline{M}(X)_\infty = \overline{M}_{0,3}(X) /\!\!/ G$$

Zero volume limit

studied by Gonzalez-Woodward

A ∞-*vortex* is a map $u : \mathbb{P}^1 \to X$ with $\int_{\mathbb{P}}^1 u^* \Phi \, \mathrm{Vol}_{\mathbb{P}^1} = 0$

Stable ∞-vortices allow sphere bubbling

$\overline{M}(X)_\infty$ moduli space of stable ∞-vortices

G acts on $\overline{M}_{0,3}(X)$ and $\overline{M}(X)_\infty$ is the pre-symplectic quotient

$$\overline{M}(X)_\infty = \overline{M}_{0,3}(X) /\!\!/ G$$

Theorem: $\overline{M}(X)_\infty$ is a compactification of $\overline{M}(P, X)_\epsilon, \epsilon \to \infty$.

Proof of compactness

Say (A_α, u_α) is a sequence of ϵ_α vortices with $\epsilon_\alpha \to \infty$

Proof of compactness

Say (A_α, u_α) is a sequence of ϵ_α vortices with $\epsilon_\alpha \to \infty$

Assume bounded energy and bounded first derivative

Proof of compactness

Say (A_α, u_α) is a sequence of ϵ_α vortices with $\epsilon_\alpha \to \infty$

Assume bounded energy and bounded first derivative

X compact implies $\|F_{A_\alpha}\|_{L^2} < C\epsilon_\alpha^{-1}$.

Proof of compactness

Say (A_α, u_α) is a sequence of ϵ_α vortices with $\epsilon_\alpha \to \infty$

Assume bounded energy and bounded first derivative

X compact implies $\|F_{A_\alpha}\|_{L^2} < C\epsilon_\alpha^{-1}$.

$\mathbb{P}^1$ simply connected, Uhlenbeck compactness implies $\|A_\alpha\|_{L^2_1} < C\epsilon_\alpha^{-1}$ after gauge

Proof of compactness

Say (A_α, u_α) is a sequence of ϵ_α vortices with $\epsilon_\alpha \to \infty$

Assume bounded energy and bounded first derivative

X compact implies $\|F_{A_\alpha}\|_{L^2} < C\epsilon_\alpha^{-1}$.

$\mathbb{P}^1$ simply connected, Uhlenbeck compactness implies $\|A_\alpha\|_{L^2_1} < C\epsilon_\alpha^{-1}$ after gauge

Vortex equation implies $\epsilon_\alpha(\mathrm{d}A_\alpha + A_\alpha \wedge A_\alpha) = u_\alpha^*\Phi \,\mathrm{Vol}_{\mathbb{P}^1}$

Proof of compactness

Say (A_α, u_α) is a sequence of ϵ_α vortices with $\epsilon_\alpha \to \infty$

Assume bounded energy and bounded first derivative

X compact implies $\|F_{A_\alpha}\|_{L^2} < C\epsilon_\alpha^{-1}$.

$\mathbb{P}^1$ simply connected, Uhlenbeck compactness implies $\|A_\alpha\|_{L^2_1} < C\epsilon_\alpha^{-1}$ after gauge

Vortex equation implies $\epsilon_\alpha(\mathrm{d}A_\alpha + A_\alpha \wedge A_\alpha) = u_\alpha^* \Phi \, \mathrm{Vol}_{\mathbb{P}^1}$

Integrate to get $\int_{\mathbb{P}^1} u_\alpha^* \Phi \, \mathrm{Vol}_{\mathbb{P}^1} \to 0$

Definition of vortex invariants

$\overline{N}(P, X)$ moduli space of vortices with framings at $z_1, \ldots, z_n \in \mathbb{P}^1$

Definition of vortex invariants

$\overline{N}(P,X)$ moduli space of vortices with framings at $z_1,\ldots,z_n \in \mathbb{P}^1$

$\overline{N}(P,X) \to X^n$ evaluation maps

Definition of vortex invariants

$\overline{N}(P, X)$ moduli space of vortices with framings at $z_1, \ldots, z_n \in \mathbb{P}^1$

$\overline{N}(P, X) \to X^n$ evaluation maps

Compose
$H_G(X)^n \to H_{G^n}(\overline{N}(P, X)) \to H(\overline{M}(P, X)) \to \mathbb{Q}$ to get *vortex invariants*

Definition of vortex invariants

$\overline{N}(P,X)$ moduli space of vortices with framings at $z_1,\ldots,z_n \in \mathbb{P}^1$

$\overline{N}(P,X) \to X^n$ evaluation maps

Compose $H_G(X)^n \to H_{G^n}(\overline{N}(P,X)) \to H(\overline{M}(P,X)) \to \mathbb{Q}$ to get *vortex invariants*

Integration requires *virtual fundamental class*

Definition of vortex invariants

$\overline{N}(P, X)$ moduli space of vortices with framings at $z_1, \ldots, z_n \in \mathbb{P}^1$

$\overline{N}(P, X) \to X^n$ evaluation maps

Compose $H_G(X)^n \to H_{G^n}(\overline{N}(P, X)) \to H(\overline{M}(P, X)) \to \mathbb{Q}$ to get *vortex invariants*

Integration requires *virtual fundamental class*

Foundations somewhat unelegant, unsatisfactory

Definition of vortex invariants

$\overline{N}(P, X)$ moduli space of vortices with framings at $z_1, \ldots, z_n \in \mathbb{P}^1$

$\overline{N}(P, X) \to X^n$ evaluation maps

Compose $H_G(X)^n \to H_{G^n}(\overline{N}(P, X)) \to H(\overline{M}(P, X)) \to \mathbb{Q}$ to get *vortex invariants*

Integration requires *virtual fundamental class*

Foundations somewhat unelegant, unsatisfactory

Conjectures

Conj 1: For $\epsilon \gg 0, \alpha \in H_G(X)^n$, $\langle\alpha\rangle_\epsilon = \langle\alpha\rangle_X^G$, the "invariant part" of Givental's invariants.

Conjectures

Conj 1: For $\epsilon \gg 0, \alpha \in H_G(X)^n$, $\langle \alpha \rangle_\epsilon = \langle \alpha \rangle_X^G$, the "invariant part" of Givental's invariants.

Conj 2: For $\epsilon^{-1} \gg 0$, $\langle \alpha \rangle_\epsilon = \langle Q\kappa_G(\alpha) \rangle_{X/G}$ where $Q\kappa_G$ is Ziltener's (conjectural) quantum Kirwan map

Conjectures

Conj 1: For $\epsilon \gg 0, \alpha \in H_G(X)^n$, $\langle \alpha \rangle_\epsilon = \langle \alpha \rangle_X^G$, the "invariant part" of Givental's invariants.

Conj 2: For $\epsilon^{-1} \gg 0$, $\langle \alpha \rangle_\epsilon = \langle Q\kappa_G(\alpha) \rangle_{X/G}$ where $Q\kappa_G$ is Ziltener's (conjectural) quantum Kirwan map

Conj 3: In between, have wall-crossing $\langle \alpha \rangle_{\epsilon_1} - \langle \alpha \rangle_{\epsilon_0} = \sum_{\epsilon \in (\epsilon_0, \epsilon_1), \psi \in \mathfrak{g}} (\# W_\psi / \# W) \langle \alpha \rangle_{X^\psi, X, (\mathfrak{g}/\mathfrak{g}_\psi)^{\mathbb{C}}, TX/TX^\psi, \epsilon}$

Conjectures

Conj 1: For $\epsilon \gg 0, \alpha \in H_G(X)^n$, $\langle\alpha\rangle_\epsilon = \langle\alpha\rangle_X^G$, the "invariant part" of Givental's invariants.

Conj 2: For $\epsilon^{-1} \gg 0$, $\langle\alpha\rangle_\epsilon = \langle Q\kappa_G(\alpha)\rangle_{X/G}$ where $Q\kappa_G$ is Ziltener's (conjectural) quantum Kirwan map

Conj 3: In between, have wall-crossing $\langle\alpha\rangle_{\epsilon_1} - \langle\alpha\rangle_{\epsilon_0} = \sum_{\epsilon\in(\epsilon_0,\epsilon_1),\psi\in\mathfrak{g}}(\#W_\psi/\#W)\langle\alpha\rangle_{X^\psi,X,(\mathfrak{g}/\mathfrak{g}_\psi)^{\mathbb{C}},TX/TX^\psi,\epsilon}$

Wall-crossing terms count vortices in X^ψ, twisted by Euler classes of index bundles $(\mathfrak{g}/\mathfrak{g}_\psi)^{\mathbb{C}}$ and TX/TX^ψ, and allowing sphere bubbling in X

Quantum Non-Abelian localization conjecture

$\langle Q\kappa_G(\alpha)\rangle_{X/G} = \langle\alpha\rangle_X^G + \sum_{\epsilon\in(0,\infty)}$ wall-crossing terms.

Quantum Non-Abelian localization conjecture

$\langle Q\kappa_G(\alpha)\rangle_{X/G} = \langle \alpha\rangle_X^G + \sum_{\epsilon\in(0,\infty)}$ wall-crossing terms.

Not really Quantum, Non-Ab, or Local :)

Quantum Non-Abelian localization conjecture

$\langle Q\kappa_G(\alpha)\rangle_{X/G} = \langle \alpha \rangle_X^G + \sum_{\epsilon \in (0,\infty)}$ wall-crossing terms.

Not really Quantum, Non-Ab, or Local :)

Theorem-in-progress: $\langle \alpha \rangle_{X,\epsilon} = \langle \alpha \rangle_X^G + \sum_{\epsilon' > \epsilon}$ wall-crossing terms for any $\epsilon > 0$

Quantum Non-Abelian localization conjecture

$\langle Q\kappa_G(\alpha)\rangle_{X/G} = \langle\alpha\rangle_X^G + \sum_{\epsilon\in(0,\infty)}$ wall-crossing terms.

Not really Quantum, Non-Ab, or Local :)

Theorem-in-progress: $\langle\alpha\rangle_{X,\epsilon} = \langle\alpha\rangle_X^G + \sum_{\epsilon'>\epsilon}$ wall-crossing terms for any $\epsilon > 0$

In other words, we believe we have proved Conjectures 1 and 3.

quantum Martin conjecture

motivation is the quantum Martin conjecture of Bertram, Ciocan-Fontanine, and Kim

quantum Martin conjecture

- motivation is the quantum Martin conjecture of Bertram, Ciocan-Fontanine, and Kim
- Hori-Vafa: conjectured relation between GW invariants of Grassmannians and products of projective spaces

quantum Martin conjecture

motivation is the quantum Martin conjecture of Bertram, Ciocan-Fontanine, and Kim

Hori-Vafa: conjectured relation between GW invariants of Grassmannians and products of projective spaces

Bertram et al: prove Hori-Vafa and conjectured $\langle \kappa_G(\alpha) \rangle_{X/G, d_G} = (\#W)^{-1} \sum_{d_T \mapsto d_G} \langle \kappa_T(\alpha) \rangle_{X/T, (\mathfrak{g}/\mathfrak{t})_{\mathbb{C}}}$.

quantum Martin conjecture

motivation is the quantum Martin conjecture of Bertram, Ciocan-Fontanine, and Kim

Hori-Vafa: conjectured relation between GW invariants of Grassmannians and products of projective spaces

Bertram et al: prove Hori-Vafa and conjectured $\langle \kappa_G(\alpha) \rangle_{X/G, d_G} = (\# W)^{-1} \sum_{d_T \mapsto d_G} \langle \kappa_T(\alpha) \rangle_{X/T, (\mathfrak{g}/\mathfrak{t})_{\mathbb{C}}}$.

Right-hand side are twisted GW invariants

quantum Martin conjecture

- motivation is the quantum Martin conjecture of Bertram, Ciocan-Fontanine, and Kim
- Hori-Vafa: conjectured relation between GW invariants of Grassmannians and products of projective spaces
- Bertram et al: prove Hori-Vafa and conjectured $\langle \kappa_G(\alpha) \rangle_{X/G, d_G} = (\# W)^{-1} \sum_{d_T \mapsto d_G} \langle \kappa_T(\alpha) \rangle_{X/T, (\mathfrak{g}/\mathfrak{t})_{\mathbb{C}}}$.
- Right-hand side are twisted GW invariants
- Ciocan-Fontanine, Kim, Sabbah, conj higher genus version

quantum Martin conjecture

- motivation is the quantum Martin conjecture of Bertram, Ciocan-Fontanine, and Kim
- Hori-Vafa: conjectured relation between GW invariants of Grassmannians and products of projective spaces
- Bertram et al: prove Hori-Vafa and conjectured $\langle \kappa_G(\alpha) \rangle_{X/G, d_G} = (\# W)^{-1} \sum_{d_T \mapsto d_G} \langle \kappa_T(\alpha) \rangle_{X/T, (\mathfrak{g}/\mathfrak{t})_{\mathbb{C}}}$.
- Right-hand side are twisted GW invariants
- Ciocan-Fontanine, Kim, Sabbah, conj higher genus version

quantum Martin conjecture: partial results

Theorem-in-progress: version of quantum Martin for any $\epsilon > 0$

quantum Martin conjecture: partial results

Theorem-in-progress: version of quantum Martin for any $\epsilon > 0$

Proof: Induction on $\dim(G)$ (for a slight generalization of the formula), Martin's argument for $\epsilon = \infty$, and method of continuity

quantum Martin conjecture: partial results

Theorem-in-progress: version of quantum Martin for any $\epsilon > 0$

Proof: Induction on $\dim(G)$ (for a slight generalization of the formula), Martin's argument for $\epsilon = \infty$, and method of continuity

Quantum non-abelian localization in general would imply genus zero quantum Martin with Kirwan replaced by quantum Kirwan