The main result of this paper is

Theorem 1 (Theorem A)**.** *Let $X^n \hookrightarrow \mathbb{P}^N$ be a smooth, linearly normal, complex algebraic variety of degree ≥ 2. Let R_X denote the **X-resultant** (the Cayley-Chow form of X). Let $\triangle_{X\times\mathbb{P}^{n-1}}$ denote the **X-hyperdiscriminant** of f format (n-1) (the defining polynomial for the dual of $X \times \mathbb{P}^{n-1}$ in the Segre embedding). Then there are norms such that the Mabuchi-energy restricted to the Bergman metrics is given as follows:*

$$\nu_\omega(\phi_\sigma) = \deg(R_X)\log\frac{\|\sigma\cdot\triangle_{X\times\mathbb{P}^{n-1}}\|^2}{\|\triangle_{X\times\mathbb{P}^{n-1}}\|^2} - \deg(\triangle_{X\times\mathbb{P}^{n-1}})\log\frac{\|\sigma\cdot R_X\|^2}{\|R_X\|^2}$$

The proof of this theorem consists of 3 steps.

1 Step 1

1.1 Jet bundle from Gauss map

$$\begin{array}{rcl} F : X & \longrightarrow & Gr(n,\mathbb{P}^N) \\ x & \mapsto & \mathbb{T}_x X \end{array}$$

Under any local complex coordinate $\{z_1,\cdots,z_n\}$, $X \subset \mathbb{P}^N$ is given by

$$(z_1,\cdots,z_n) \mapsto [1, Z_1(z),\cdots,Z_N(z)]$$

F is given by

$$(z_1,\ldots,z_n) \mapsto \mathrm{Span}_{\mathbb{C}}\left\{\begin{array}{llll} v_0 = (1, & Z_1(z), & \cdots, & Z_N(z)) \\ v_1 = (0, & \frac{\partial Z_1}{\partial z_1}, & \cdots, & \frac{\partial Z_N}{\partial z_1}) \\ & \cdots & \cdots & \\ v_n = (0, & \frac{\partial Z_1}{\partial z_n}, & \cdots, & \frac{\partial Z_N}{\partial z_n}) \end{array}\right\}$$

Lemma 1.

$$F^*\omega_{Gr} = (n+1)\omega_{FS} - Ric(\omega_{FS})$$

Definition 1 (Jet bundle)**.**

$$(J(O(1))^\vee, h_{J(O(1))}) = F^*(\mathcal{U}, h_{\mathbb{C}^{N+1}})$$

By the above Lemma, the first Chern class of jet bundle gives the first Chern class of TX. This gives some motivation for considering the jet bundle.

1.2 Incidence diagram

From the incidence diagram, one sees that the dual of X is closely related to the jet bundle.

$I_X = \{(x,\mathbb{H}) \in X\times(\mathbb{P}^N)^\vee; \mathbb{T}_x X \subset \mathbb{H}\} =$ zero locus of a section of $\pi_2^*\mathcal{O}(1)\otimes\pi_1^* J(\mathcal{O}(1)) \subset X\times(\mathbb{P}^N)^\vee$

$I_\triangle = \{(\mathbb{T},\mathbb{H}) \in Gr(n,\mathbb{P}^N)\times(\mathbb{P}^N)^\vee; \mathbb{T}\subset\mathbb{H}\} =$ zero locus of a section of $\pi_2^*\mathcal{O}(1)\otimes\pi_1'^*\mathcal{U}^\vee \subset Gr(n,\mathbb{P}^N)\times(\mathbb{P}^N)^\vee$

$$\begin{array}{ccccc} X & \xleftarrow{\pi_1} & I_X & \xrightarrow{\pi_2} & (\mathbb{P}^N)^\vee \\ {\scriptstyle F}\downarrow & & {\scriptstyle F\times id}\downarrow & & \|\downarrow \\ Gr(n,\mathbb{P}^N) & \xleftarrow{\pi_1'} & I_\triangle & \xrightarrow{\pi_2} & (\mathbb{P}^N)^\vee \end{array}$$

Definition 2.

$$X^\vee = \pi_2(I_X) = \{\mathbb{H} \in \mathbb{P}^{N\vee}; \exists x \in X, s.t. \mathbb{T}_x X \subset \mathbb{H}\} \subset \mathbb{P}^{N\vee}$$

Assumption 1. *$\pi_2|_{I_X} : I_X \to X^\vee$ is birational, $X^\vee = \{\Delta_X = 0\} \subset (\mathbb{P}^N)^\vee$ is a hypersurface.*

Lemma 2.

$$\deg(X^\vee) = \int_X c_n(J(\mathcal{O}_X(1))) \tag{1}$$

Lemma 3. *Via Poincaré Duality and G-invariance.*

$$\pi'_{1*}\pi_2^*\omega_{FS}^N = c_{n+1}(\mathcal{U}^\vee, h_{\mathbb{C}^{N+1}})$$

1.3 Bott-Chern form and complex Hessian formula

Using incidence diagram and properties of Bott-Chern form, we can transform the integration of Bott-Chern form on $X^\vee$ to integration of Bott-Chern form on X up to a $\partial\bar{\partial}$ closed function.

For any compactly supported smooth (m-1,m-1)-form η, $m = \dim G$. In the following calculation, $\int_{GI_X}$ is the bridge connecting $\int_{GX^\vee}$ and $\int_{GX}$,

$$\begin{aligned}
& N\int_G \eta \wedge \partial\bar{\partial} \int_0^1 dt \int_{X^\vee} \dot{\Phi}_\sigma \omega_{FS(\mathbb{P}^\vee)}^{N-1} = \int_G \partial\bar{\partial}\eta \wedge \int_{X^\vee} BC(\mathcal{O}_{\mathbb{P}^\vee}(1), c_1^N; h, h(\sigma)) \\
= & \int_{GX^\vee} \eta \wedge \partial\bar{\partial}\, BC(\mathcal{O}_{\mathbb{P}^\vee}(1), c_1^N; h, h(\sigma)) = \int_{GX^\vee} \eta \wedge c_1^N(\mathcal{O}_{\mathbb{P}^\vee}(1), h(\sigma)) \\
= & \int_{GI_X} \eta \wedge \pi_2^* c_1^N(\mathcal{O}_{\mathbb{P}^\vee}(1), h(\sigma)) \\
= & \int_{GX} \eta \wedge \pi_{1*}\pi_2^* c_1^N(\mathcal{O}_{\mathbb{P}^\vee}(1), h(\sigma)) = \int_{GX} \eta \wedge (GF)^* \pi'_{1*}\pi_2^* c_1^N(\mathcal{O}_{\mathbb{P}^\vee}(1), h(\sigma)) \\
= & \int_{GX} \eta \wedge c_{n+1}((GF)^*\mathcal{U}^\vee, h(\sigma)) = \int_{GX} \eta \wedge c_{n+1}(J(\mathcal{O}(1)), h_{J(\mathcal{O}(1))}(\sigma)) \\
= & \int_{GX} \eta \wedge \partial\bar{\partial} BC(J(\mathcal{O}(1)), c_{n+1}; h, h(\sigma)) = \int_G \eta \wedge \partial\bar{\partial} \int_X BC(J(\mathcal{O}(1)), c_{n+1}; h, h(\sigma))
\end{aligned}$$

So on $G = SL(N+1, \mathbb{C})$,

$$N\partial\bar{\partial} \int_0^1 dt \int_{X^\vee} \dot{\Phi}_\sigma \omega_{FS(\mathbb{P}^\vee)}^{N-1} = \partial\bar{\partial} \int_0^1 dt \int_X BC(J(\mathcal{O}(1)), c_{n+1}; h, h(\sigma))$$

Remark 1 (Tian's argument)**.** *If we have Log Polynomial growth for the integral on the write hand side, we will get*

$$N \int_0^1 dt \int_{X^\vee} \dot{\Phi}_\sigma \omega_{FS(\mathbb{P}^\vee)}^{N-1} = \int_0^1 dt \int_X BC(J(\mathcal{O}(1)), c_{n+1}; h, h(\sigma)) \tag{2}$$

This should be true in general. For the K-energy case considered in this paper, one can verify this directly in Step 3. See (11).

2 Step 2

The goal of this step is to express the Bott-Chern form on jet bundle in terms of Bott-Chern form on TX. For this, we need the exact sequence for jet bundle and Griffith's formula for the curvature of vector bundle in exact sequence. Then one also needs to prove a metric splitting theorem for the exact sequence.

2.1 Exact sequence for Jet bundle

$$0 \to T^*X \otimes \mathcal{O}(1) \to J_X(\mathcal{O}(1)) \to \mathcal{O}(1) \to 0$$

Equivalently,

$$\begin{array}{ccccccc} 0 & \to \mathcal{O}(-1) & \xrightarrow{f} & J_X(\mathcal{O}(1))^\vee & \xrightarrow{g} & \mathcal{O}(-1)\otimes TX & \to 0 \\ & \| & & \| & & \| & \\ & \mathcal{S} & & \mathcal{E} & & \mathcal{Q} & \end{array} \tag{3}$$

2.2 Griffith Formula and calculation of 2nd fundamental form

Split orthogonal frames of $J(\mathcal{O}(1))^\vee$

$$e_0 = v_0 = (1, Z_1, \cdots, Z_n)$$

$$e_i = v_i - \frac{\langle v_i, e_0\rangle}{|e_0|^2} e_0$$

$$g(e_i) = g(v_i) = e_0 \otimes \frac{\partial}{\partial z_i}$$

Under $\{e_0, e_1, \cdots, e_n\}$, there is a **differentiable** isomorphism

$$\mathcal{E} = \mathcal{S} \oplus \mathcal{S}^\perp \xlongequal{id \oplus g} \mathcal{S} \oplus \mathcal{Q}$$

Under the split orthogonal frames, one writes

$$D^{\mathcal{E}} = \begin{pmatrix} D^{\mathcal{S}} & \beta \\ \alpha & D^{\mathcal{Q}} \end{pmatrix}$$

The 2nd fundamental form $\alpha \in C^\infty\left(T^{*1,0}X \otimes Hom(\mathcal{S}, \mathcal{Q})\right)$ is of (1,0) type.

$$\alpha(e_0) = g\left(\left(\frac{\partial}{\partial z_i} e_0\right)^\perp\right) = g(e_i) = e_0 \otimes \frac{\partial}{\partial z_i}$$

So

$$\alpha = \sum_i dz_i \otimes \frac{\partial}{\partial z_i} \tag{4}$$

In particular, α is holomorphic.

$\beta = -\alpha^* \in C^\infty\left(T^{*(0,1)} \otimes Hom(\mathcal{Q}, \mathcal{S})\right)$.

$$\beta\left(\frac{\partial}{\partial \bar{z}_j}\right)\left(e_0 \otimes \frac{\partial}{\partial z_i}\right) = \left\langle \frac{\partial}{\partial \bar{z}_j} e_i, \frac{e_0}{1+|Z|^2}\right\rangle e_0 = -(\omega_{FS})_{i\bar{j}} e_0$$

$$\beta\left(e_0 \otimes \frac{\partial}{\partial z_i}\right) = -\omega_{i\bar{j}} d\bar{z}_j \otimes e_0$$

Proposition 1. *Griffith's Formula:*

$$F^{\mathcal{E}} = (D^{\mathcal{E}})^2 = \begin{pmatrix} F^{\mathcal{S}} + \beta \circ \alpha & D^{\mathcal{S}} \circ \beta + \beta \circ D^{\mathcal{Q}} \\ D^{\mathcal{Q}} \circ \alpha + \alpha \circ D^{\mathcal{S}} & F^{\mathcal{Q}} + \alpha \circ \beta \end{pmatrix}$$

In the jet bundle case

$$\beta \circ \alpha(e_0) = \beta(dz_i \otimes e_0 \otimes \frac{\partial}{\partial z_i}) = -dz_i \otimes (-\omega_{i\bar{j}} d\bar{z}_j \otimes e_0) = \omega \otimes e_0$$

$$\alpha \circ \beta(e_0 \otimes \frac{\partial}{\partial z_i}) = -\alpha(\omega_{i\bar{j}} d\bar{z}_j \otimes e_0) = \omega_{i\bar{j}} d\bar{z}_j \wedge dz_k \otimes \frac{\partial}{\partial z_k} \otimes e_0 = -\omega_{i\bar{j}} dz_k \otimes d\bar{z}_j \otimes \left(e_0 \otimes \frac{\partial}{\partial z_k} \right)$$

$$S_i^k := (\alpha \circ \beta)_i^k = -\omega_{i\bar{j}} dz_k \otimes d\bar{z}_j = -dz_k \otimes d\bar{z}_i$$

Proposition 2.

$$F^{\mathcal{E}} = \begin{pmatrix} 0 & 0 \\ 0 & -\omega_{FS}|_X \otimes I_{T_X^{1,0}} + F_{\omega_{FS}}^{T_X^{1,0}} + S \end{pmatrix} =: \begin{pmatrix} 0 & 0 \\ 0 & \tilde{F} \end{pmatrix} \tag{5}$$

2.3 Bott-Chern form of Jet bundle

Theorem 2.

$$BC(J(\mathcal{O}(1))^\vee, c_{n+1}; h, h(\sigma)) = -\dot{\phi}_\sigma c_n(J(\mathcal{O}(1))^\vee, h(\sigma)) \tag{6}$$

Proof. By formula (5)

$$\begin{aligned} BC(J(\mathcal{O}(1))^\vee, c_{n+1}; h, h(\sigma)) &= \frac{d}{db} \det \left(F^{\mathcal{E}} + bH^{-1}\dot{H} \right) \\ &= \frac{d}{db} \left(-b\dot{\phi}_\sigma \det(\tilde{F} + b\tilde{H}^{-1}\dot{\tilde{H}}) \right) \\ &= -\dot{\phi}_\sigma \det(\tilde{F}) = -\dot{\phi}_\sigma c_n(J(\mathcal{O}(1))^\vee, h(\sigma)) \end{aligned}$$

□

Corollary 1.

$$BC(J(\mathcal{O}(1)), c_{n+1}; h, h(\sigma)) = \dot{\phi}_\sigma c_n(J(\mathcal{O}(1), h(\sigma))$$

2.4 Metric splitting of exact sequence (3)

Theorem 3.

$$c(J(\mathcal{O}(1))^\vee, h_{\mathbb{C}^{N+1}}) = c(TX \otimes \mathcal{O}(-1), \omega_{FS} \otimes h_{FS}^*) \cdot c(\mathcal{O}(-1), h_{FS}^*)$$

Proof.

$$F^{\mathcal{S} \oplus \mathcal{Q}} = \begin{pmatrix} -\omega_{FS}|_X & 0 \\ 0 & -\omega_{FS}|_X \otimes I_{T_X^{1,0}} + F_{\omega_{FS}}^{T_X^{1,0}} \end{pmatrix}$$

To show the Chern forms split, one only needs to show

$$Tr((F^{\mathcal{E}})^k) = Tr((F^{\mathcal{S} \oplus \mathcal{Q}})^k) \tag{7}$$

for $1 \leq k \leq n$. This is because $Tr(A^k)$ generates all invariant polynomials.

By Lemma 4, it's easy to show both sides of (7) equal to

$$2(-1)^k \omega^k + \sum_{i=1}^{k} \binom{k}{i} (-1)^{k-i} \omega^{k-i} Tr(F^i)$$

□

Lemma 4.

$$Tr((F+S)^k) = Tr(F^k) - \omega^k$$

Combining Theorem 3 and Corollary 1, one achieves the goal of expressing the Bott-Chern form on jet bundle in terms of Bott-Chern form on TX and $\mathcal{O}(1)$.

3 Step 3

The step is to replace X by $X\times\mathbb{P}^{n-1}$. This has two uses. One is to make sure the dual of $X\times\mathbb{P}^{n-1}$ is of codimension one. The other use is to eliminate the extra curvature terms so that only Ricci curvature is preserved.

3.1 Pass to Hyper-discriminant: $X \rightsquigarrow X\times\mathbb{P}^{n-1}$

Claim 1. *For $X\times\mathbb{P}^{n-1}$, The Assumption 1 is always satisfied. This is called Cayley's trick.*

$$0\to\mathcal{O}_{\mathbb{P}^{n-1}}\to\mathcal{O}_{\mathbb{P}^{n-1}}(1)^{\oplus n}\to T\mathbb{P}^{n-1}\to 0$$

$$0\leftarrow\mathcal{O}_{\mathbb{P}^{n-1}}(1)\leftarrow\mathcal{O}_{\mathbb{P}^{n-1}}^{\oplus n}\leftarrow T^*\mathbb{P}^{n-1}\otimes\mathcal{O}_{\mathbb{P}^{n-1}}(1)\leftarrow 0 \tag{8}$$

Lemma 5. *Metric splitting for the exact sequence (8):*

$$c(\mathcal{O}_{\mathbb{P}^{n-1}}(1),h_{FS})\cdot c(T\mathbb{P}^{n-1}\otimes\mathcal{O}_{\mathbb{P}^{n-1}}(1),g_{FS}^*\otimes h_{FS}) = 1$$

By the above Lemma and Theorem 3,

$$\begin{aligned} c(J(\mathcal{O}(1,1)),h_1) &= c(TX\otimes\mathcal{O}(1,1),h_2)\cdot c(T\mathbb{P}^{n-1}\otimes\mathcal{O}(1,1),h_3)\cdot c(\mathcal{O}(1,1),h_4) \\ &= c(TX\otimes\mathcal{O}(1,1),h_2)(1+\omega_{FS(\mathbb{P}^N)}|_X)^n \end{aligned}$$

Modulo unitary transformation, let

$$\frac{\sqrt{-1}}{2\pi}R^{T^*X\otimes\mathcal{O}_X(1)} = \mathrm{diag}(x_1+y,\cdots,x_n+y)$$

$$\frac{\sqrt{-1}}{2\pi}R^{\mathcal{O}_{\mathbb{P}^{n-1}}(1)} = \omega_{FS(\mathbb{P}^{n-1})} = z$$

Then

$$\begin{aligned} c(TX\otimes\mathcal{O}(1,1),h_2) &= (1+x_1+y+z)\cdots(1+x_n+y+z) = z^{n-1}(n+x_1+\cdots+x_n+ny) \\ &= \omega^{n-1}_{FS(\mathbb{P}^n)}(n-Ric(\omega)+n\omega) \end{aligned}$$

Theorem 4.

$$\begin{aligned} c_{2n-1}(J(\mathcal{O}(1,1)),h_1) &= \{z^{n-1}(n-Ric(\omega)+n\omega)(1+\omega)^n\}_{(2n-1)} \\ &= z^{n-1}(n-Ric(\omega)+n\omega)(n\omega^{n-1}+\omega^n) \\ &= z^{n-1}(n(n+1)\omega^n - nRic(\omega)\wedge\omega^{n-1}) \end{aligned}$$

3.2 Log Polynomial Growth of K-energy

This extra discussion is to make sure one can drop the $\partial\bar{\partial}$ in formula (2).

Lemma 6. *For any $\sigma \in SL(N+1, \mathbb{C})$, the holomorphic bisectional curvature $S_{i\bar{j}k\bar{l}}$ of ω_σ satisfies:*

$$h^{i\bar{j}} h^{k\bar{l}} S_{i\bar{j}k\bar{l}} \leq 2 \tag{9}$$

$h = g_\sigma$ is the metric associated with Kähler form ω_σ.

Proof. For any point $P \in X$, choose coordinate such that $h_{i\bar{j}} = \delta_{ij}$. By Gauss' formula:

$$\tilde{R}(\partial_i, \overline{\partial_i}, \partial_j, \overline{\partial_j}) = S_{i\bar{i}j\bar{j}} + |II(\partial_i, \partial_j)|^2$$

where $\tilde{R}$ is the curvature of Fubini-Study metric of ambient $\mathbb{P}^N$. $\tilde{R}$ satisfies:

$$\tilde{R}_{i\bar{j}k\bar{l}} = \tilde{g}(\partial_i, \overline{\partial_j})\tilde{g}(\partial_k, \overline{\partial_l}) + \tilde{g}(\partial_i, \overline{\partial_l})\tilde{g}(\partial_k, \overline{\partial_j}) = h_{i\bar{j}} h_{k\bar{l}} + h_{i\bar{l}} h_{k\bar{j}}$$

So under normal coordinate of ω_σ,

$$S_{i\bar{i}j\bar{j}} \leq \tilde{R}(\partial_i, \overline{\partial_i}, \partial_j, \overline{\partial_j}) = \delta_{ii}\delta_{jj} + \delta_{ij}^2 \leq 2$$

□

Let $f = tr_\omega \omega_\sigma$ and Δ be the complex Laplacian associated with Kähler metric ω, $R_{k\bar{j}}$ be the Ricci curvature of reference metric ω and $S_{i\bar{j}k\bar{l}}$ be the curvature of Kähler metric ω_σ. Let ∇ be the gradient operator associated with g, then

$$\begin{aligned}
\Delta \log f &= \frac{\Delta f}{f} - \frac{|\nabla f|^2_\omega}{f^2} \\
&\geq \frac{g^{i\bar{l}} g^{k\bar{j}} R_{k\bar{l}} h_{i\bar{j}}}{f} - \frac{g^{i\bar{j}} g^{k\bar{l}} S_{i\bar{j}k\bar{l}}}{f} \\
&= \frac{\sum_i \mu_i^{-2} R_{i\bar{i}}}{\sum_i \mu_i^{-1}} - \frac{\sum_{i,j} \mu_i^{-1}\mu_j^{-1} S_{i\bar{i}j\bar{j}}}{\sum_i \mu_i^{-1}} \\
&\geq -C_1 - 2\sum_i \mu_i^{-1} = -C_1 - C_2 f
\end{aligned} \tag{10}$$

where $-C_1$ is the lower bound of $Ric(\omega)$. In the 3rd equality in (10), for any fixed point $P \in X$, we chose a coordinate near P such that $h_{i\bar{j}} = \delta_{ij}$, $\partial_k h_{i\bar{j}} = 0$. We can assume g is also diagonalized so that

$$g_{i\bar{j}} = \mu_i \delta_{ij}$$

For the last inequality in (10), we used the inequality (9).

So

$$\Delta(\log f + \lambda\phi_\sigma) \geq -C_1 - C_2 f + \lambda tr_\omega(\omega_\sigma - \omega) = (\lambda - C_2)f - (C_1 + n\lambda) = C_3 f - C_4$$

for some constants $C_3 > 0$, $C_4 > 0$, if we choose λ to be sufficiently large. So at the maximum point P of the function $\log f + \lambda\phi_\sigma$, we have

$$0 \geq \Delta(\log f + \lambda\phi_\sigma)(P) \geq C_3 f(P) - C_4$$

So

$$f(P) = tr_\omega(\omega_\sigma)(P) \leq \frac{C_4}{C_3} = C_5$$

So for any point $x \in X$, we have

$$tr_\omega \omega_\sigma(x) \le C_5 e^{-\lambda(\phi_\sigma(x)-\phi_\sigma(P))} \le C_5 e^{\lambda \operatorname{osc}(\phi_\sigma)}$$

So

$$\omega_\sigma \le C_5 e^{\lambda \operatorname{osc}(\phi_\sigma)}\omega$$

Since $Osc(\phi_\sigma)$ has log polynomial growth,

$$\log \frac{\omega_\sigma^n}{\omega^n} \le n \log C_5 + n\lambda \operatorname{osc}(\phi_\sigma)$$

has log polynomial upper growth. The lower bound of K-energy follows from convexity of Logarithmic function. So by Claim 1, one gets

Proposition 3. *The functional*

$$-\int_0^1 dt \int_X n\dot{\phi}_\sigma(Ric(\omega_\sigma) - Ric(\omega_0)) \wedge \omega_\sigma^{n-1} = \int_X \log \frac{\omega_\sigma^n}{\omega_0^n}\omega_\sigma^n$$

has log polynomial growth as a function on $SL(N+1, \mathbb{C})$*.*

Substitute this into (6) and (2), one gets

Theorem 5 (Hyper-discriminant part in the K-energy)**.**

$$N\int_0^1 dt \int_{(X\times\mathbb{P}^{n-1})^\vee} \dot{\Phi}_\sigma \omega_{FS(\mathbb{P}^\vee)}^{N-1} = \int_0^1 dt \int_X \dot{\phi}_\sigma(n(n+1)\omega_\sigma^n - nRic(\omega_\sigma)\wedge\omega_\sigma^{n-1}) \qquad (11)$$

3.3 Other Ingredient and Main Formula

Lemma 7 (Tian)**.**

$$\log \frac{\|\sigma \cdot R_X\|^2}{\|R_X\|^2} = (n+1)\int_0^1 \int_X \dot{\phi}_\sigma \omega_\sigma^n$$

$$\log \frac{\|\sigma \cdot \triangle_{X\times\mathbb{P}^{n-1}}\|^2}{\|\triangle_{X\times\mathbb{P}^{n-1}}\|^2} = N\int_0^1 dt \int_{(X\times\mathbb{P}^{n-1})^\vee} \dot{\Phi}_\sigma \omega_{FS(\mathbb{P}^\vee)}^{N-1}$$

Lemma 8.

$$\deg(\triangle_{X\times\mathbb{P}^{n-1}}) = \deg((X\times\mathbb{P}^{n-1})^\vee) = \int_{X\times\mathbb{P}^{n-1}} c_{2n-1}(J(\mathcal{O}(1,1))) = (n(n+1) - n\mu)V$$

while

$$\deg(R_X) = (n+1)d$$

Theorem 6 (Main Formula)**.**

$$\begin{aligned}
& -\int_0^1 \int_X \dot{\phi}_\sigma(S(\omega) - \underline{S})\omega^n = -\int_X \dot{\phi}_\sigma(nRic(\omega) - n\mu)\wedge\omega^{n-1} \\
= \; & -(n(n+1)-n\mu)\int_0^1 dt \int_X \dot{\phi}_\sigma \omega^n + \int_0^1 dt \int_X \dot{\phi}_\sigma(n(n+1)\omega - nRic(\omega))\wedge\omega^{n-1} \\
= \; & -\frac{(n(n+1)-n\mu)d}{(n+1)d}\int_0^1 dt \int_X (n+1)\dot{\phi}_\sigma\omega^n + \int_0^1 dt \int_0^1 \int_{X^\vee} N\dot{\Phi}_\sigma \omega_{FS^\vee}^{N-1} \\
= \; & -\frac{\deg(\triangle_{X\times\mathbb{P}^{n-1}})}{\deg(R_X)}\log\frac{\|\sigma\cdot R_X\|^2}{\|R_X\|^2} + \log\frac{\|\sigma\cdot\triangle_{X\times\mathbb{P}^{n-1}}\|^2}{\|\triangle_{X\times\mathbb{P}^{n-1}}\|^2}
\end{aligned}$$

This is just Theorem 1.