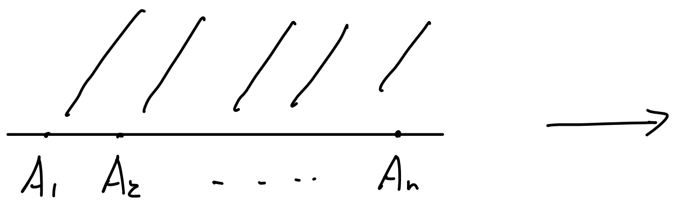


$$S(z) = \int_0^z \frac{d\zeta}{(\zeta-A_1)^{\beta_1}\cdots(\zeta-A_n)^{\beta_n}},$$

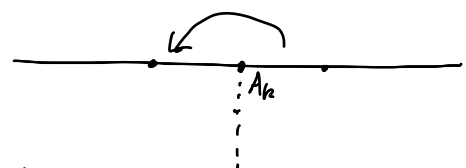
$$(\zeta - A_j)^{\beta_j} = e^{\beta_j \cdot \log(\zeta - A_j)}$$

$$\begin{cases} (x-A_j) & \zeta = x > A_j \\ \underset{\displaystyle e^{(\log(A_j-x)+i\pi)\beta_j}}{\underset{\shortparallel}{(A_j-x)^{\beta_j} \cdot e^{i\beta_j \pi}}} & \zeta = \underset{\mathbb{R}}{\underset{\cap}{x}} < A_j \end{cases}$$

$0 < \beta_i < 1.$ $\quad \sum_{j=1}^{n} \beta_j = 2.$

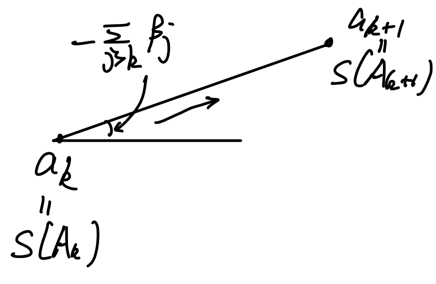
$A_k < x < A_{k+1}$

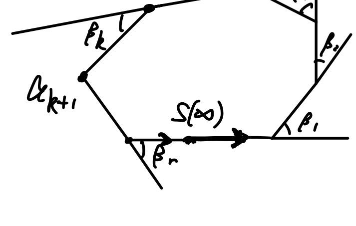
$$S'(x) = \frac{1}{(x-A_1)^{\beta_1}\cdots(x-A_n)^{\beta_n}}$$

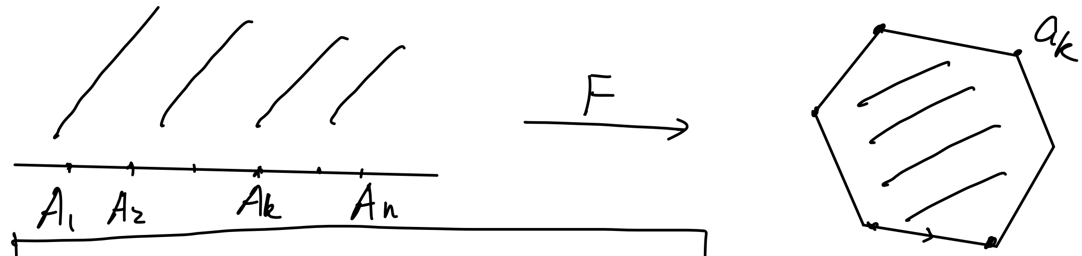


$$= \frac{1}{\prod_{j\le k}(x-A_j)^{\beta_j}} \cdot \frac{1}{\prod_{j>k}(A_j-x)^{\beta_j} \cdot e^{i\beta_j\pi}}$$

$$\arg(S'(x)) = \begin{cases} -\left(\sum_{j>k}\beta_j\right)\pi & A_k < x < A_{k+1} \\ -\sum_{j>0}\beta_j = \underset{0}{\underset{\shortparallel}{-2\pi}}, & x < A_1 \\ 0. & x > A_n \end{cases}$$







$$\boxed{\text{Thm: } F(z) = C_1 S(z) + C_2}$$

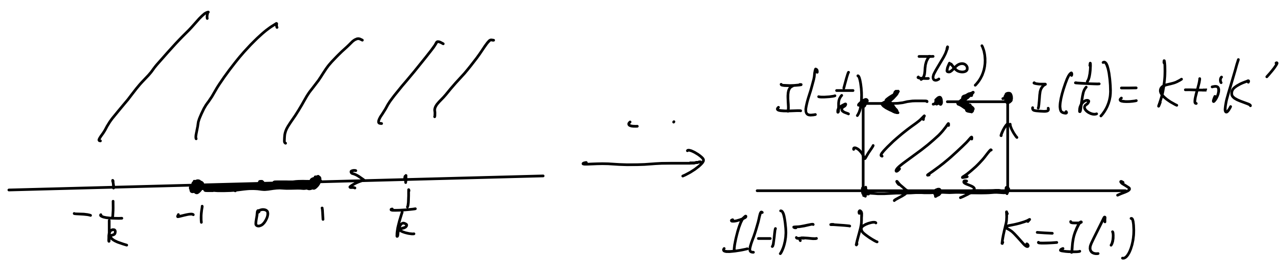
Ex: $I(z) = \int_0^z \frac{d\zeta}{[(1-\zeta^2)(1-k^2\zeta^2)]^{\frac{1}{2}}}$ $\qquad 0<k<1.$

$$\| $$

$$[(\zeta+1)(\zeta-1)\cdot(\zeta-\tfrac{1}{k})\cdot(\zeta+\tfrac{1}{k})]^{\frac{1}{2}}$$

$\beta_j = \frac{1}{2}$, $\beta_j \pi = \frac{\pi}{2}$

$\pi - \beta\pi = \frac{\pi}{2}$



$-1 < x < 1$, $\quad \int_0^x \frac{dt}{[(1-t^2)\cdot(1-k^2t^2)]^{\frac{1}{2}}}$

$(x^2-1)^{\frac{1}{2}}\cdot(-i).$

$$\|$$

$1 < x < \frac{1}{k}$

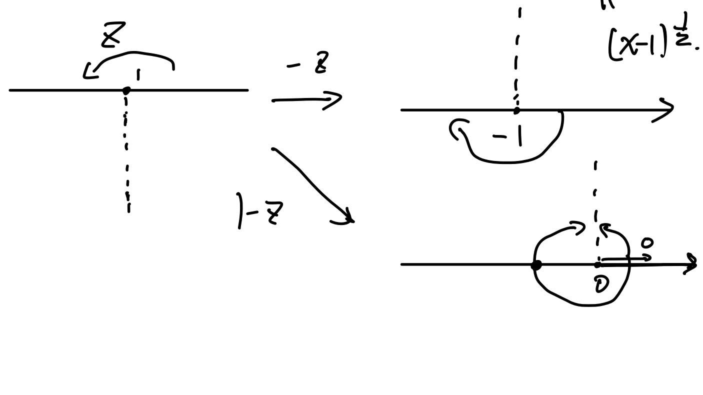
$(1-x^2)^{\frac{1}{2}} = ((-1)\cdot(x^2-1))^{\frac{1}{2}} = (x^2-1)^{\frac{1}{2}}(-1)^{\frac{1}{2}}$

$$\|$$

$(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}$ $\qquad 1-k^2t^2>0.$

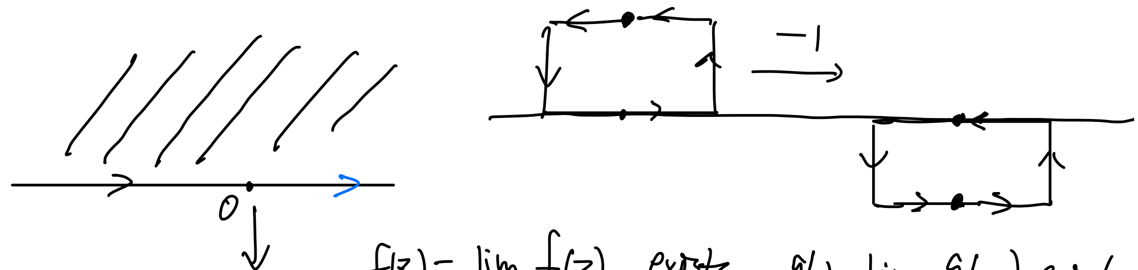
$$\|$$

$(x-1)^{\frac{1}{2}}\cdot e^{-i\cdot\pi\cdot\frac{1}{2}} = -i\cdot(x-1)^{\frac{1}{2}}$



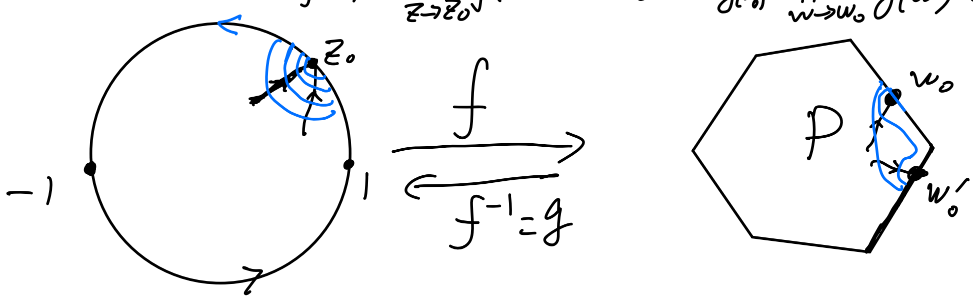
$$I(x) = \underbrace{\int_0^1 \frac{dt}{[(1-t^2)(1-k^2t^2)]^{\frac{1}{2}}}}_{K} + \underbrace{\int_1^x \frac{dt}{(t^2-1)^{\frac{1}{2}}(-i)(1-k^2t^2)^{\frac{1}{2}}}}_{i \int_1^x \frac{dt}{(t^2-1)^{\frac{1}{2}}(1-k^2t^2)^{\frac{1}{2}}}}$$

∞

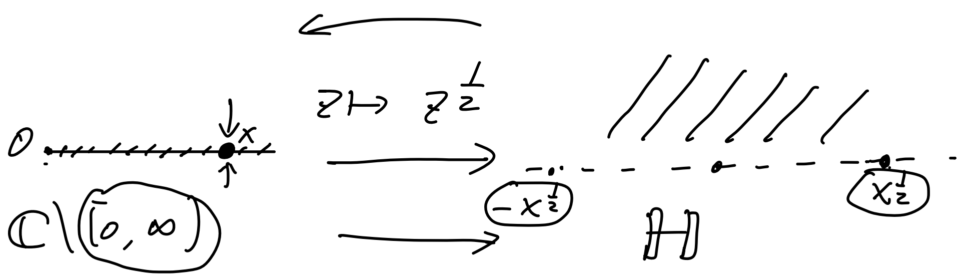


$f(z_0) = \lim_{z\to z_0} f(z)$ exists $\quad g(w_0) = \lim_{w\to w_0} g(w)$ exists

Thm:

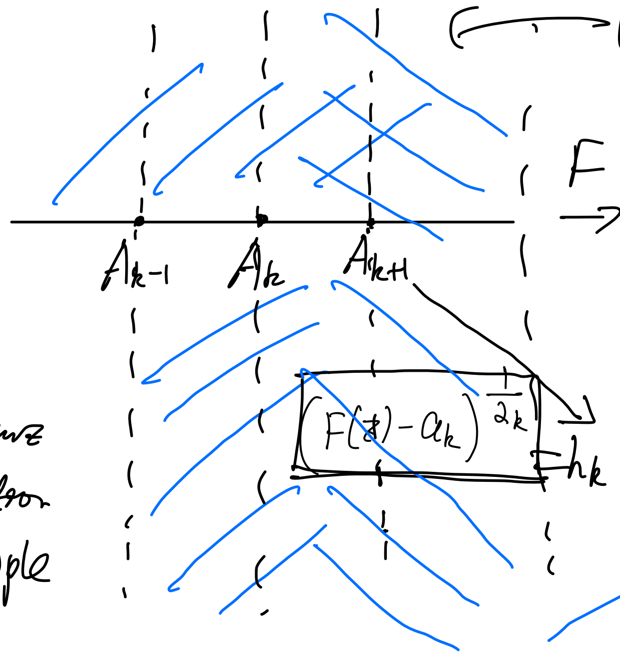


f extends to a continuous bijection to $f: \overline{\mathbb{D}} \to \overline{P}$

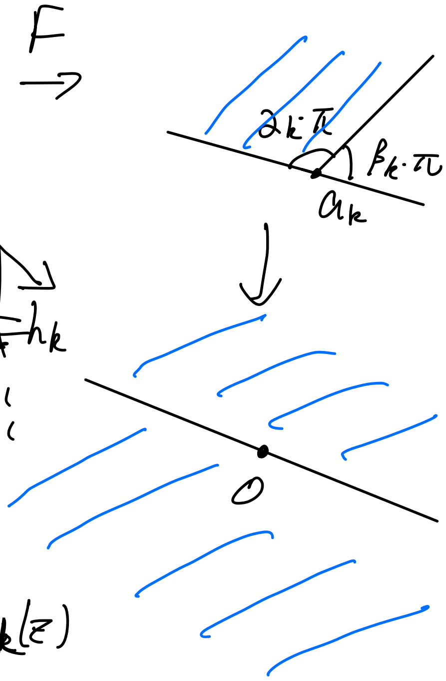


$F(z)$

$\longleftarrow (F(z)-a_{k+1})^{\frac{1}{\alpha_{k+1}}} = h_{k+1}$



Schwarz reflection principle



$$h_k = (F(z)-a_k)^{\frac{1}{\alpha_k}} = (z-A_k)\cdot g_k(z)$$

$$h_k^{\alpha_k} = F(z)-a_k \Rightarrow \alpha_k \cdot h_k^{\alpha_k-1} h_k' = F'(z). \qquad h_k' \neq 0.$$

$$1-\alpha_k=\beta_k, \qquad \alpha_k \overset{-\beta_k}{\overset{\shortparallel}{(\alpha_k-1)}} h_k^{\alpha_k-2} h_k'^2 + \alpha_k h_k^{\alpha_k-1} h_k'' = F''(z).$$

$$\frac{F''(z)}{F'(z)} = -\frac{\beta_k}{z-A_k} + E_k(z). \sim -\beta_k \cdot h_k^{-1} \cdot h_k' + h_k'' \cdot h_k'^{-1}$$

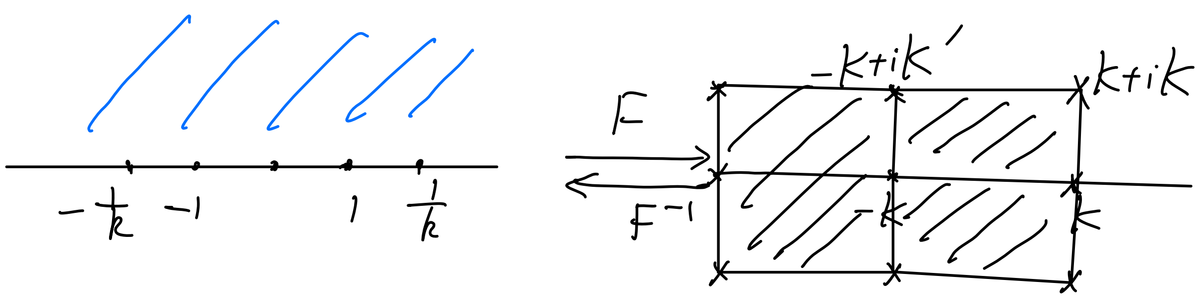
- meromorphic fct. on $\mathbb{C}$ with poles at A_k of order 1.
- converges to 0 as $z \to \infty$.

$$\Rightarrow \boxed{\frac{F''(z)}{F'(z)} = -\frac{\beta_1}{z-A_1} - \cdots - \frac{\beta_n}{z-A_n}}$$

$$\Rightarrow \quad (\log F')' = \left(+ \log \frac{1}{\prod (z-A_i)^{\beta_i}} \right)'$$

$$\Rightarrow \quad F' = C_1 \frac{1}{\prod_i (z-A_i)^{\beta_i}}$$

$$\Rightarrow \quad \boxed{F = C_1 \int_0^z \frac{d\zeta}{\prod_i (\zeta - A_i)^{\beta_i}} + C_2 .}$$



$$F(z) = \int_0^z \frac{d\zeta}{\left[(1-\zeta^2)\cdot(1-k^2\zeta^2)\right]^{\frac{1}{2}}}$$

$$\rightsquigarrow \quad \mathbb{C} \xleftarrow{\ sn(z)\ } \mathbb{C}$$

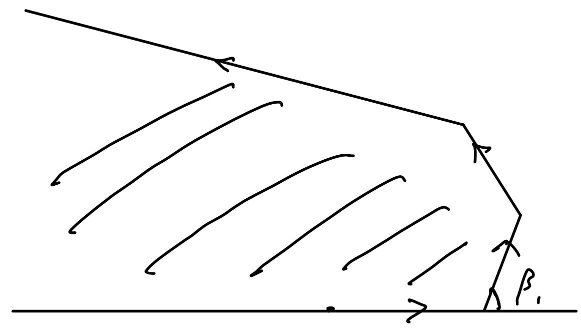
doubly periodic function.

meromorphic function

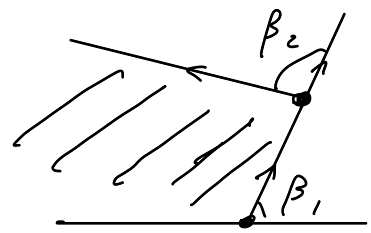
$\boxed{\text{elliptic function}}$

$\sum \beta_i = 2$

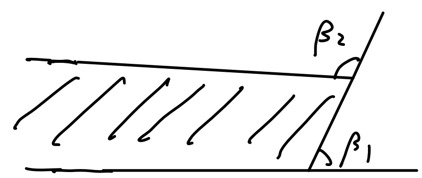
$\sum \beta_i < 2$



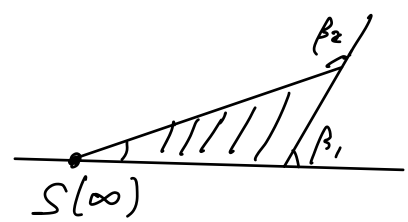
$0 < \beta_1 + \beta_2 < 1$



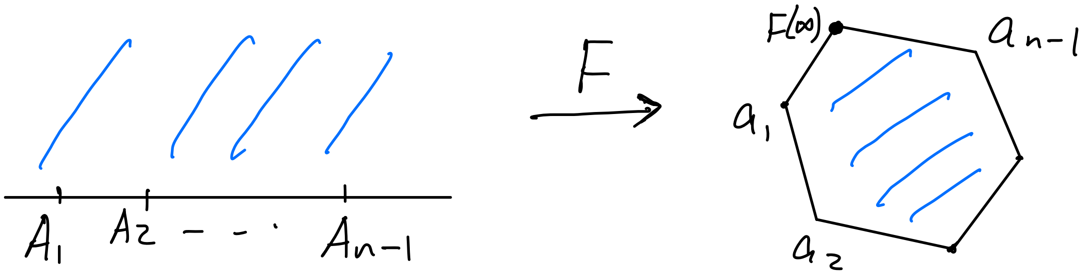
$\beta_1 + \beta_2 = 1$



$1 < \beta_1 + \beta_2 < 2$



$$S(z) = \int_0^z \frac{d\zeta}{(\zeta - A_1)^{\beta_1}(\zeta - A_2)^{\beta_2}}$$



$$F(z) = C_1 \cdot \int_0^z \frac{d\zeta}{(\zeta - A_1)^{\beta_1} \cdots (\zeta - A_{n-1})^{\beta_{n-1}}} + C_2$$

$$1 < \beta_1 + \cdots + \beta_{n-1} < 2$$

<u>Ex</u>: $S(z) = \int_0^z \frac{d\zeta}{(1-\zeta^2)^{\frac{1}{2}}}$ $\qquad \beta_1 = \beta_2 = \frac{1}{2}$. $\beta_1 + \beta_2 = 1$