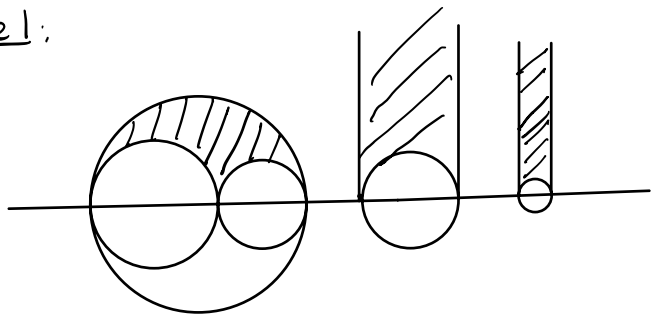
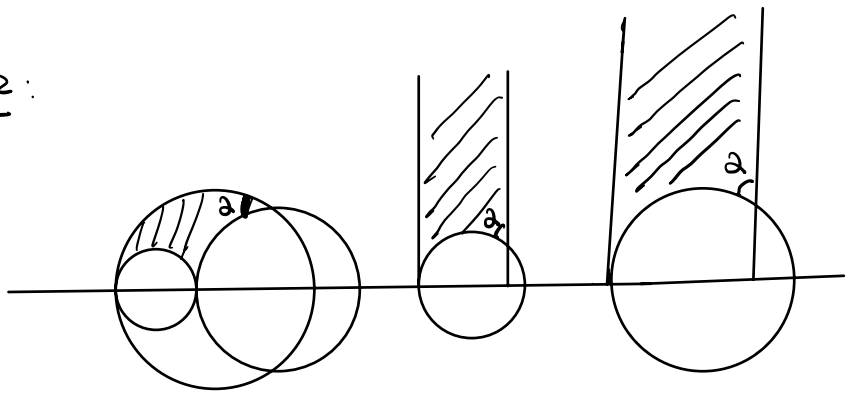
Area of non-Euclidean triangles.

Case 1:



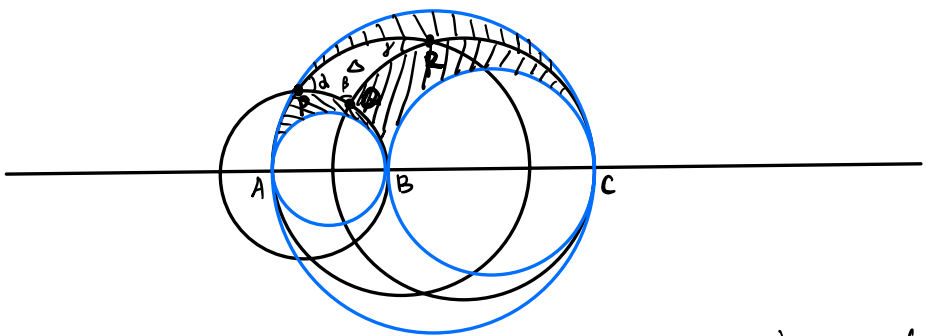
$\text{Area}(\Delta_i) = \pi$.

Case 2:



$$\text{Area} = \pi - \alpha = \int_\Delta \frac{dx\,dy}{y^2}$$

General case:

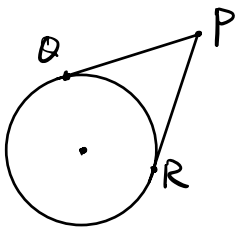


$$\text{Area}(\Delta_{PQR}) = \text{Area}(\Delta_{ABC}) - \text{Area}(\Delta_{PAB}) - \text{Area}(\Delta_{BQC}) - \text{Area}(\Delta_{CAR})$$

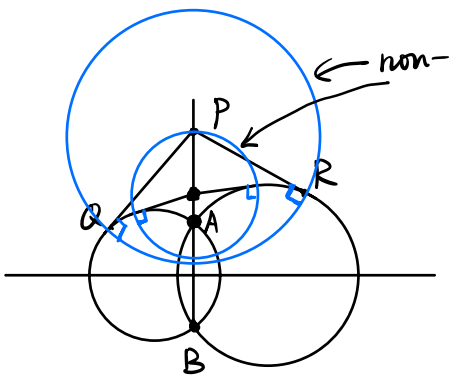
$$= \pi - (\pi - (\pi - \alpha)) - (\pi - (\pi - \beta)) - (\pi - (\pi - \gamma))$$

$$= \pi - (\alpha + \beta + \gamma).$$

Steiner : power of a point w.r.t. a circle.



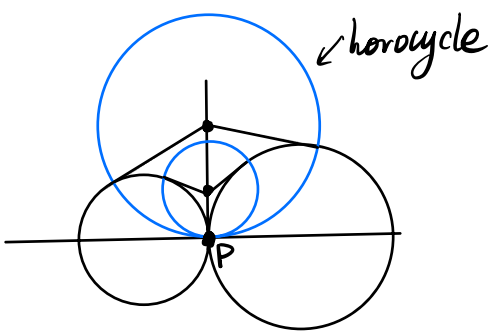
$|PQ| = |PR|$.



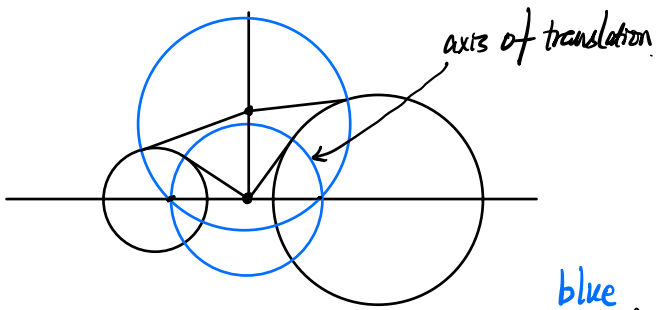
$|PQ|^2 = |PA| \cdot |PB| = |PR|^2$

$\Rightarrow |PQ| = |PR|$.

rotation around A



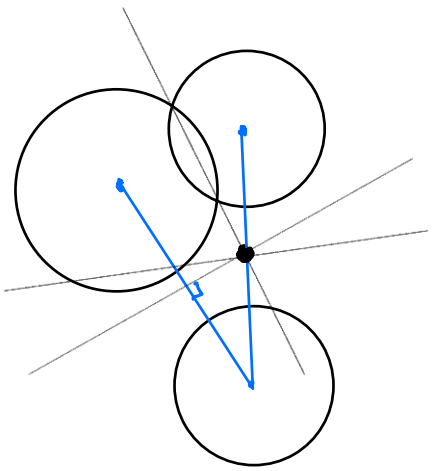
limit rotation around P



blue

translation (along partial circles)

Given any three circles, there is a unique point that has the same powers w.r.t. these circles:



radi=0 case: