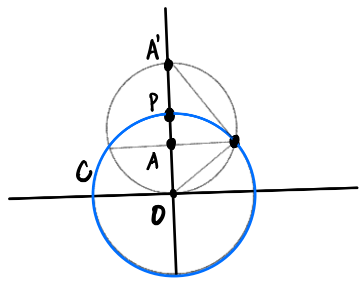
3 reflection thm in non-Euclidean geometry.

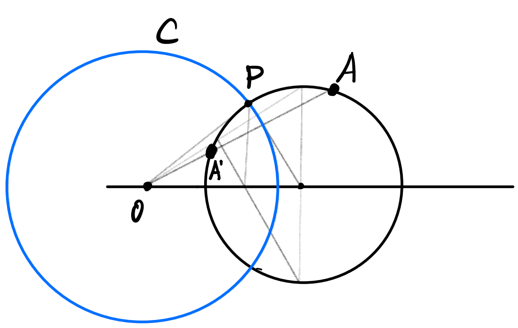
- Given any two points A, A' there is a unique equidistant line = set of points that have the same distance to A and A'

Moreover, the reflection in this line maps A to A'



$$|OP|^2 = |OA| \cdot |OA'|$$

$\Rightarrow$ reflection in the "line" C maps A to A'



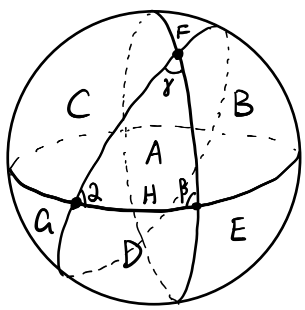
$$|OP|^2 = |OA| \cdot |OA'|$$

$\Rightarrow$ reflection in the "line" C maps A to A'.

- The same argument as in the case of $\mathbb{R}^2$ or S^2 also proves the 3 reflection thm. for the upper half plane.

Area of spherical triangles

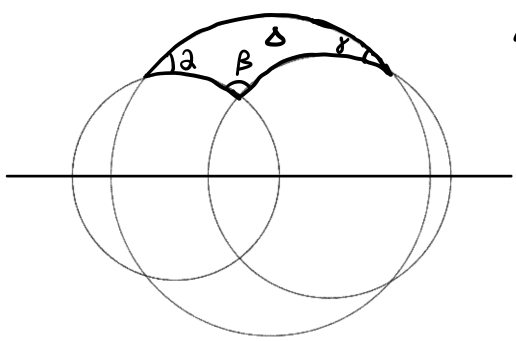
Area of sphere with radius $1 = 4\pi$



$$\left.\begin{aligned} A+B &= 4\pi\cdot\frac{\alpha}{2\pi} = 2\alpha \\ A+C &= 2\beta \\ A+D &= 2\gamma \\ A+B+C+F &= 2\pi \\ F &= D \end{aligned}\right\} \quad \begin{aligned} &\text{Area}(A) \\ &\quad \| \\ &\alpha+\beta+\gamma-\pi. \end{aligned}$$

antipodal

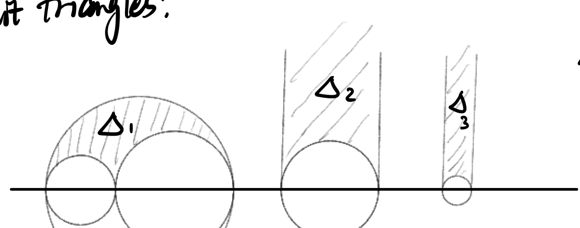
Area of non-Euclidean triangles



$$\text{Area}(\Delta) = \pi - (\alpha+\beta+\gamma).$$

$\Uparrow$ Gauss

limit triangles:



$$\text{Area}(\Delta_1) = \text{Area}(\Delta_2) = \text{Area}(\Delta_3)$$
$$\|$$
$$\pi - (0+0+0) = \pi.$$

Δ_i can be transformed to Δ_j by a linear fractional transformation

(extension of linear fractional transformation of $\mathbb{RP}^1$ that maps vertices of Δ_i to vertices of Δ_j)