3.4.1 $\begin{cases} x^2+y^2=1 \\ (x-1)^2+(y-2)^2=4 \end{cases}$ $\Leftrightarrow$ $x^2+y^2-2x-4y=-1$

$(x-1)^2+(y-2)^2 = x^2-2x+1+y^2-4y+4$

$\Rightarrow \begin{cases} x^2+y^2=1 \\ 2x+4y=2 \end{cases} \Leftrightarrow x+2y-1=0 \Leftrightarrow x=1-2y$

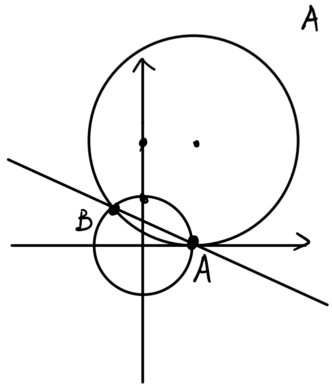
$\Rightarrow 1=(1-2y)^2+y^2=1-4y+4y^2+y^2=1-4y+5y^2$

$\Rightarrow 0=-4y+5y^2=y\cdot(5y-4) \Rightarrow y=0$ or $y=\frac{4}{5}$

$\Rightarrow \begin{cases} x=1 \\ y=0 \end{cases}$ or $\begin{cases} x=-\frac{3}{5} \\ y=\frac{4}{5} \end{cases}$

so intersection points: $\underset{A}{(1,0)}$ and $\underset{B}{\left(-\frac{3}{5}, \frac{4}{5}\right)}$.

3.4.2–3.4.3:

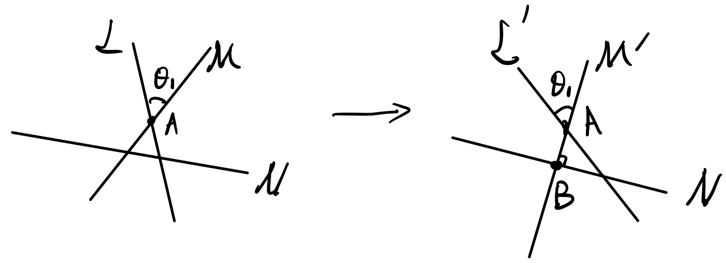


$x+2y-1$ is the equation for the line AB:

$$y-0=\frac{\frac{4}{5}-0}{-\frac{3}{5}-1}\cdot(x-1)=-\frac{1}{2}(x-1) \Leftrightarrow x+2y-1=0.$$

3.6.5 / 3.6.7 :- Show $r_N \circ r_M \circ r_L = r_N \circ r_{M'} \circ r_{L'}$ with $M' \perp N$.

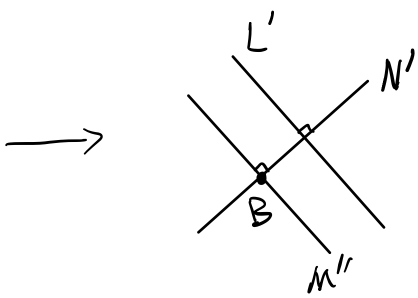
Here need to assume $M \cap L \neq \phi$. Then we can rotate the axes M and L simultaneously to get M' and L' with $M' \perp N$ and



$$r_M \circ r_L = r_{M'} \circ r_{L'}$$
$$\Downarrow$$
$$r_N \circ r_M \circ r_L = r_N \circ r_{M'} \circ r_{L'}$$

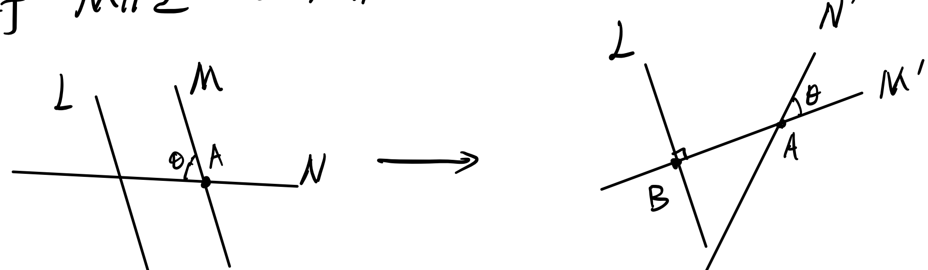
- rotate M' and N simultaneously to get M'' and N' with $M'' \parallel L'$, $N \perp M''$ and $r_N \circ r_{M'} = r_{N'} \circ r_{M''}$.

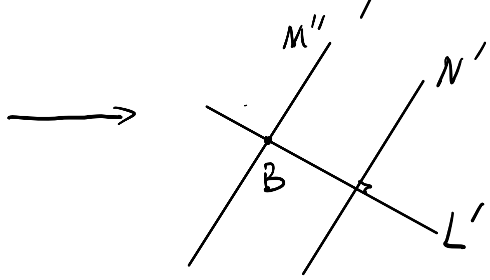
$$\Downarrow$$



$$r_N \circ r_{M'} \circ r_{L'} = \underbrace{r_{N'} \circ r_{M''} \circ r_{L'}}_{\text{a glide reflection.}}$$

If $M \parallel L$ and $N \nparallel M$, then





$r_{N'} \circ r_{M''} \circ r_{L'}$ is a glide reflection.

If $M // L // N$, then $r_N \circ r_M \circ r_L$ is just a reflection.

3.7.4 . Because a reflection reverses the orientation of $\mathbb{R}^2$ (left or right handedness)

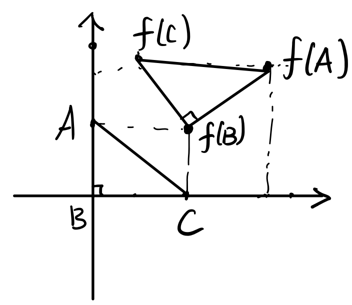
So • if an isometry reverses the orientation, then it must be a glide reflection or simply a reflection.

• In this case, the middle points of $\overline{Af(A)}$, $\overline{Bf(B)}$, $\overline{Cf(C)}$ lie on a line that is the mirror of reflection.

• If an isometry preserves the orientation, then it must be either a translation or a rotation

• An isometry f is a translation if $f(A)-A=f(B)-B=f(C)-C$.

3.7.3 Because f reverses the orientation, f must be a reflection or a glide reflection.



middle point of $\overline{A f(A)}$: $\frac{1}{2}((0,1)+(1.8,1.6))=(0.9,1.3)$

- - - of $\overline{Bf(B)}$: $\frac{1}{2}((0,0)+(1,1))=(0.5,0.5)$

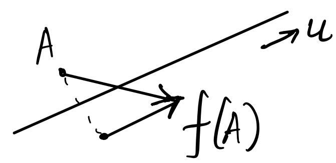
- - - of $\overline{Cf(C)}$: $\frac{1}{2}((1,0)+(0.4,1.8))=(0.7,0.9)$.

$\Rightarrow$ direction of mirror is $\frac{u}{|u|}$, $u=(1,2)$

The line of reflection has slope: $\frac{1.3-0.5}{0.9-0.5}=2=\frac{0.9-0.5}{0.7-0.5}=k'$.

slope of $Af(A)=\frac{1.6-1}{1.8-0}=\frac{0.6}{1.8}=\frac{1}{3}=k$ $\quad k\cdot k'=\frac{2}{3}\neq -1 \Rightarrow$ not a single reflection.

The translation vector of the glide reflection is given by



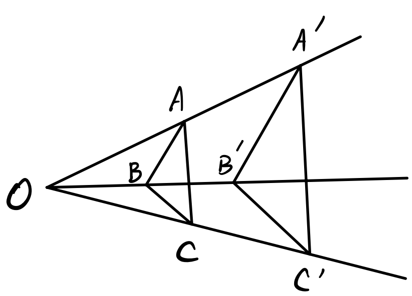
$$\mathrm{Proj}_u \overrightarrow{Af(A)} = \left(\overrightarrow{Af(A)} \cdot \frac{u}{|u|}\right)\frac{u}{|u|} = \frac{\overrightarrow{Af(A)} \cdot u}{|u|^2}\, u$$

$$= \frac{(1.8, 0.6)\cdot(1,2)}{1^2+2^2}\cdot(1,2) = \frac{3}{5}(1,2)$$

$$\mathrm{Proj}_u \overrightarrow{Bf(B)} = \frac{\overrightarrow{Bf(B)} \cdot u}{|u|^2}\, u = \frac{(1,1)\cdot(1,2)}{1^2+2^2}\cdot(1,2) = \frac{3}{5}(1,2).\ \checkmark$$

$$\mathrm{Proj}_u \overrightarrow{Cf(C)} = \frac{\overrightarrow{Cf(C)} \cdot u}{|u|^2}\, u = \frac{(-0.6, 1.8)\cdot(1,2)}{1^2+2^2}(1,2) = \frac{3}{5}(1,2)\ \checkmark.$$

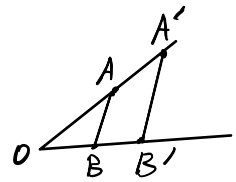
4.2.1
4.2.2



Vector Desargues Thm: $\overrightarrow{OA} = u$, $\overrightarrow{OA'} = u'$
$\overrightarrow{OB} = v$, $\overrightarrow{OB'} = v'$
$\overrightarrow{OC} = w$, $\overrightarrow{OC'} = w'$

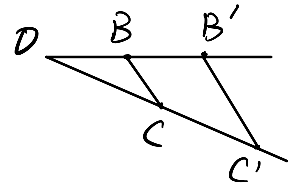
If $(v-u) \parallel (v'-u')$ and $(w-v) \parallel (w'-v')$, then $(w-u) \parallel (w'-u')$.
$(AB \parallel A'B')$ $(BC \parallel B'C')$ $(AC \parallel A'C')$

<u>Proof</u>: By Vector Thales Thm applied to



$\exists\, a \in \mathbb{R}$ s.t. $u' = au$ and $v' = a \cdot v$

By Vector Thales Thm applied to



we get $v' = av \Rightarrow w' = a \cdot w$.

So $\overrightarrow{A'C'} = w' - u' = a \cdot w - a \cdot u = a(w - u) = a \cdot \overrightarrow{AC}$

$\Rightarrow AC \,//\, A'C'$.