Tiling semiregular

3 tiles at a vertex : l-gon, m-gon, n-gon.

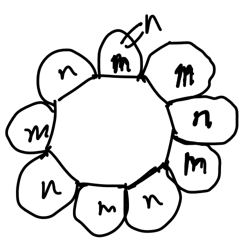
$$\frac{(l-2)\cdot\pi}{l}+\frac{(m-2)\cdot\pi}{m}+\frac{(n-2)\cdot\pi}{n}=2\pi$$

$$1-\frac{2}{l}+1-\frac{2}{m}+1-\frac{2}{n}=2$$

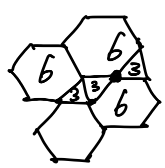
$$\Rightarrow \boxed{\frac{2}{l}+\frac{2}{m}+\frac{2}{n}=1}$$

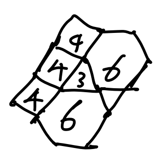
$$l=4,\ m=6 \Rightarrow \underbrace{\frac{2}{4}+\frac{2}{6}}_{\frac{1}{2}+\frac{1}{3}=\frac{5}{6}}+\frac{2}{n}=1 \Rightarrow \frac{2}{n}=1-\frac{5}{6}=\frac{1}{6} \Rightarrow n=12.$$

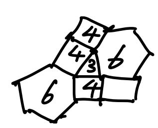
· If l is odd, then $\underline{m=n.}$



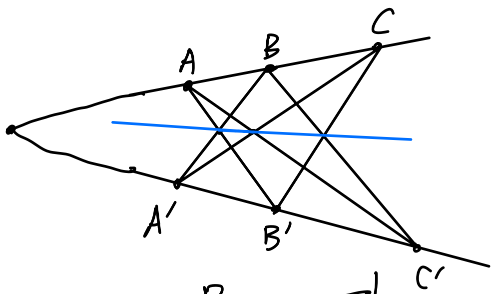
(3, 3, 6, 6)



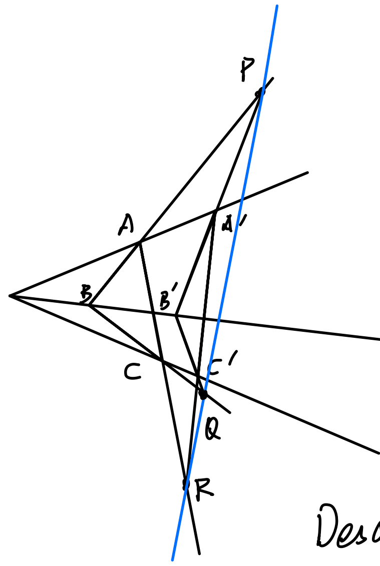




Projective Geometry



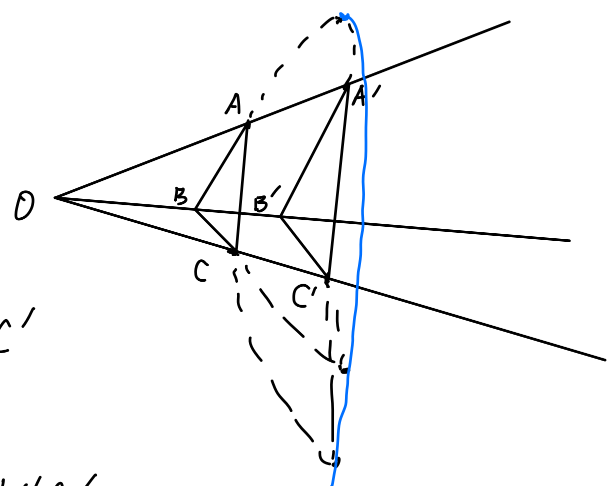
Pappus Thm

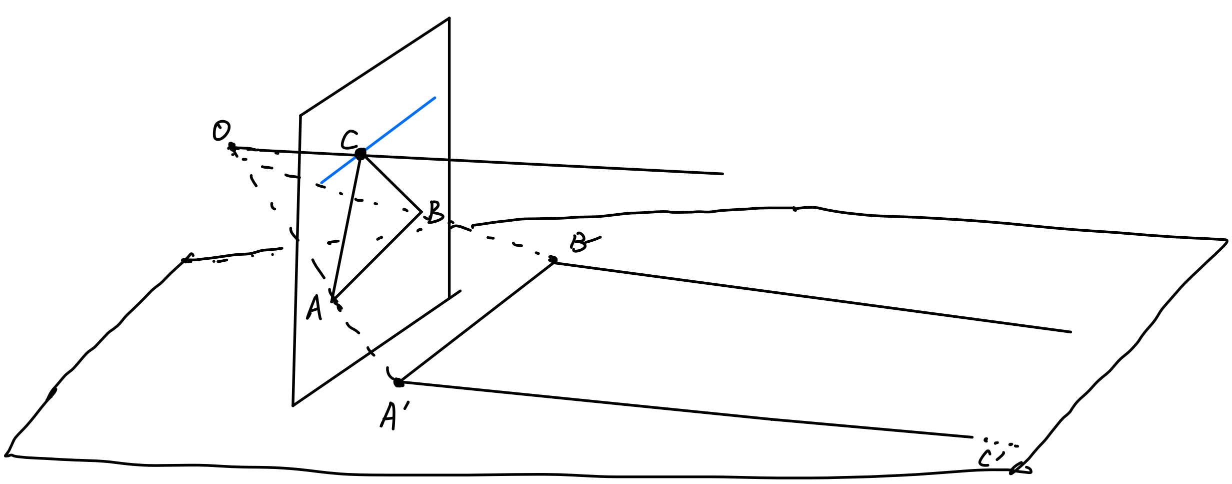


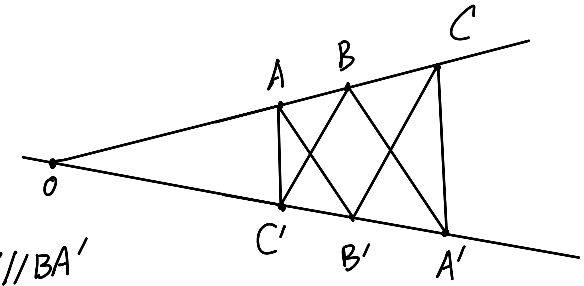
Desargues' Thm.

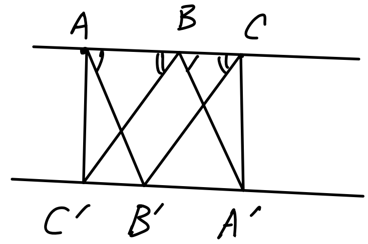
$$\begin{cases} \dfrac{|OA|}{|OA'|} = \dfrac{|OB|}{|OB'|} \overset{\text{Thales}}{\Longleftarrow} AB \parallel A'B' \\[2ex] \dfrac{|OB|}{|OB'|} = \dfrac{|OC|}{|OC'|} \Longleftarrow BC \parallel B'C' \end{cases}$$

$$\Rightarrow \frac{|OA|}{|OA'|} = \frac{|OC|}{|OC'|} \xRightarrow[\text{Thales}]{\text{Inverse}} AC \parallel A'C'$$



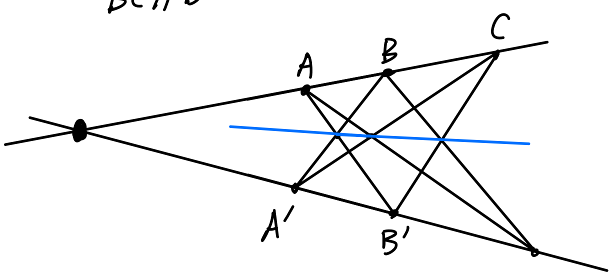




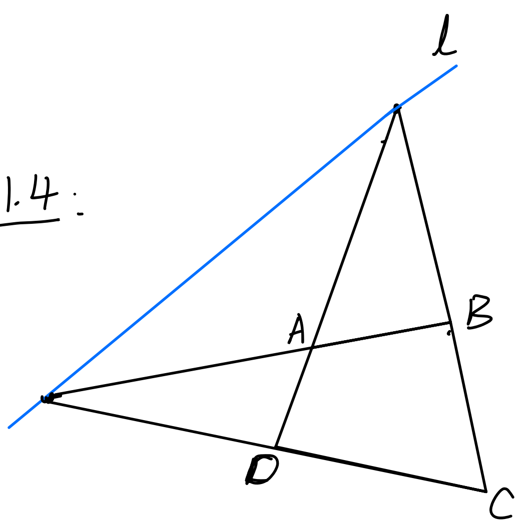


$$\left.\begin{array}{c} AB' /\!/ BA' \\ \Downarrow \\ \dfrac{|OA|}{|OB|} = \dfrac{|OB'|}{|OA'|} \\[2ex] \dfrac{|OB|}{|OC|} = \dfrac{|OC'|}{|OB'|} \\ \Uparrow \\ BC' /\!/ B'C \end{array}\right\} \Rightarrow \frac{|OA|}{|OC|} = \frac{|OC'|}{|OA'|} \Rightarrow AC' /\!/ AC.$$

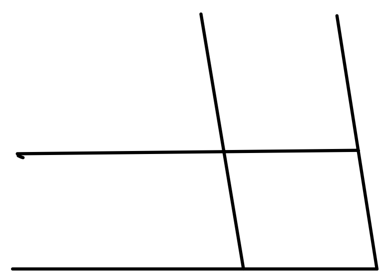
$$\begin{array}{l} |AB| = |A'B'| \\ |BC| = |B'C'| \end{array} \Rightarrow |AC| = |A'C'|$$

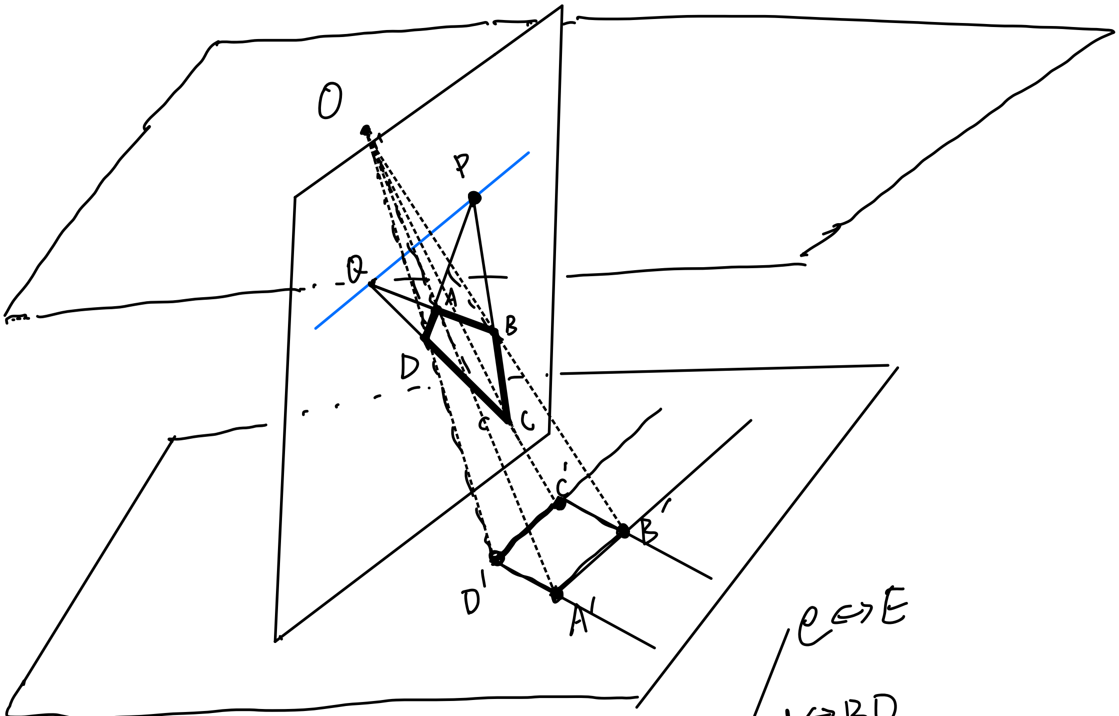


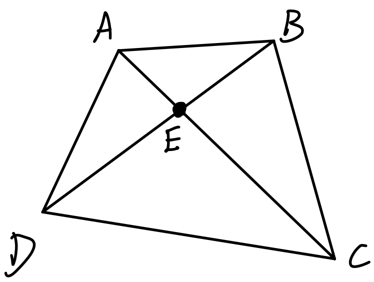
11.4:

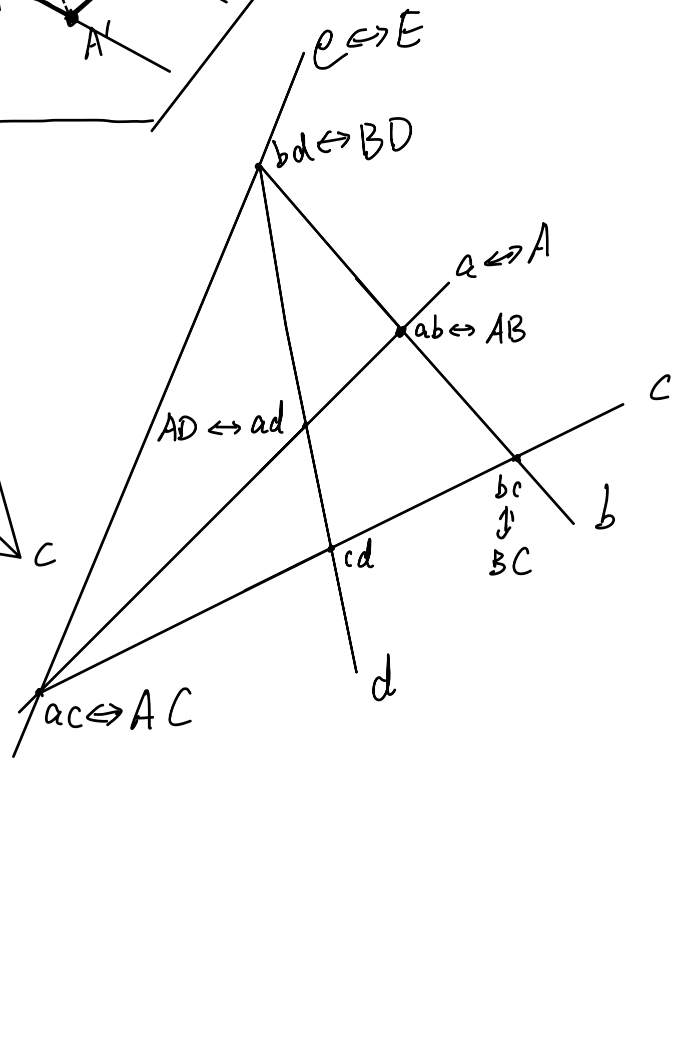


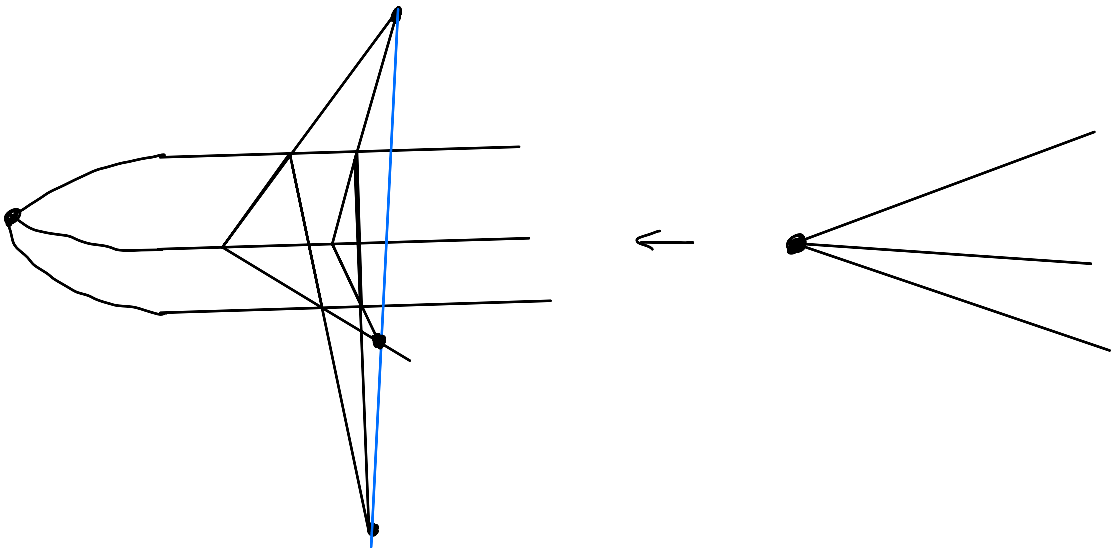
$\rightsquigarrow$

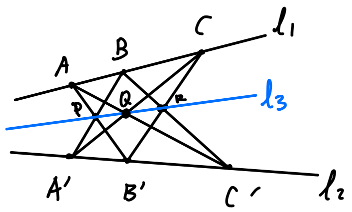


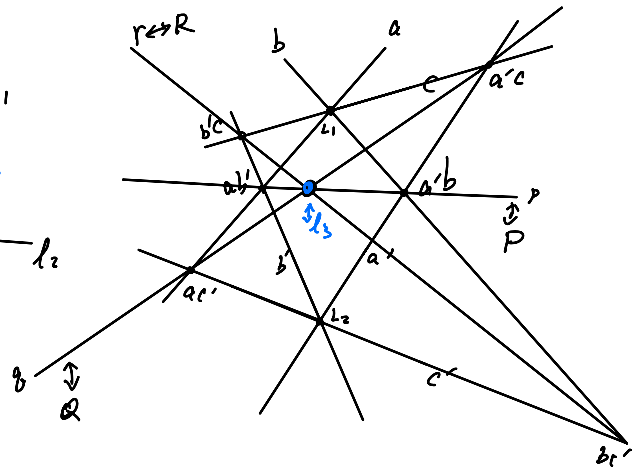

O
P
Q
A
B
D
C
c
C'
B'
D'
A'


A
B
E
D
C


e ⟺ E
bd ⟺ BD
a ⟺ A
ab ⟺ AB
AD ⟷ ad
c
bc
⇕
BC
b
cd
d
ac ⟺ AC




A
B
C
l_1
P
Q
R
l_3
A'
B'
C'
l_2


r ↔ R
b
a
c
a'c
b'c
l_1
ab'
a'b
p ↕ P
l_3
b'
a'
ac'
l_2
q ↕ Q
c'
bc'