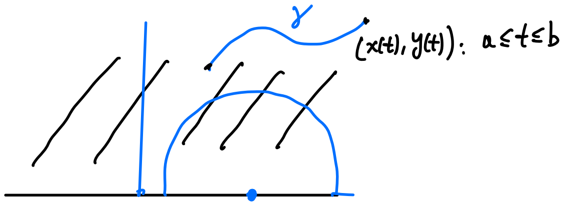
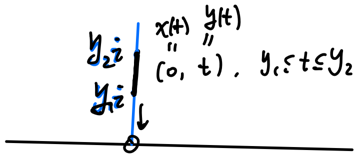
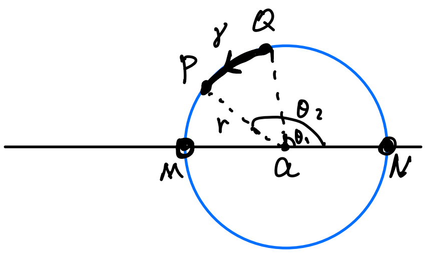


$$ds^2 = \frac{dx^2+dy^2}{y^2}$$

$$L(\gamma)=\int_a^b ds = \int_a^b \frac{\sqrt{x'(t)^2+y'(t)^2}}{y(t)}\,dt$$



$$L(\gamma)=\int_{y_1}^{y_2}\frac{\sqrt{0^2+1^2}}{t}\,dt = \ln t\Big|_{y_1}^{y_2} = \ln\frac{y_2}{y_1}$$



$$\begin{cases} x(t)=a+r\cdot\cos\theta \\ y(t)=0+r\cdot\sin\theta \end{cases}\qquad x'(t)=-r\cdot\sin\theta \qquad y'(t)=r\cdot\cos\theta$$

$$-d\cdot\cos\theta$$
$$\|$$

$$L(\gamma)=\int_{\theta_1}^{\theta_2}\frac{\sqrt{r^2\sin^2\theta+r^2\cos^2\theta}}{r\cdot\sin\theta}\,d\theta = \int_{\theta_1}^{\theta_2}\frac{d\theta}{\sin\theta} = \frac{\sin\theta\, d\theta}{\sin^2\theta}$$

$$\sin^2\theta=1-\cos^2\theta$$

$$= -\int_{\theta_1}^{\theta_2}\frac{d\cdot\cos\theta}{1-\cos^2\theta} \overset{x=\cos\theta}{=\!=} -\int_{\cos\theta_1}^{\cos\theta_2}\frac{dx}{1-x^2}$$

$$\int\frac{dx}{1-x^2}=\int\frac12\left(\frac{1}{1+x}+\frac{1}{1-x}\right)dx = \frac12\ln\left|\frac{1+x}{1-x}\right|.$$

$$= \frac12\ln\left|\frac{1+x}{1-x}\right|\Bigg|_{\cos\theta_2}^{\cos\theta_1}.$$

$$\frac{1+\cos\theta}{1-\cos\theta}=\frac{2\cdot\cos^2\frac{\theta}{2}}{2\cdot\sin^2\frac{\theta}{2}}=\cot^2\frac{\theta}{2}$$

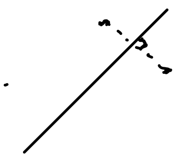
$$\frac12\ln\left|\frac{1+\cos\theta}{1-\cos\theta}\right| = \ln\cot\frac{\theta}{2}.$$

$$= \ln\cot\frac{\theta}{2}\Big|_{\theta_2}^{\theta_1}$$

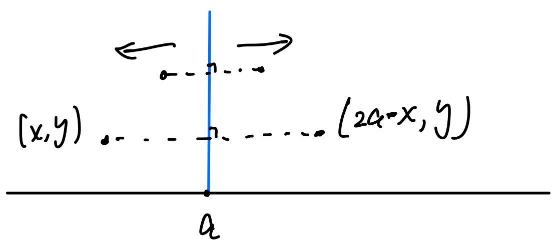
$$= \ln\frac{\cos\frac{\theta_1}{2}}{\cos\frac{\theta_2}{2}}$$

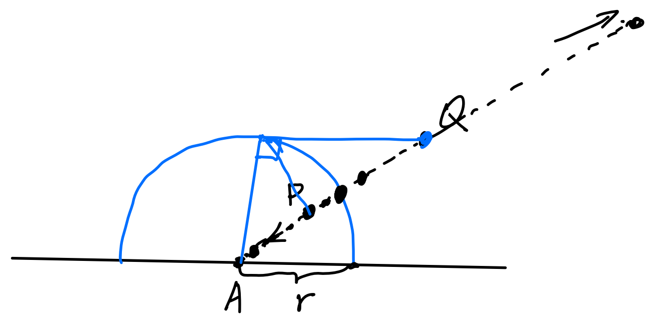
Isometry of hyperbolic upper half plane.

Euclidean : translation. rotation. reflection.

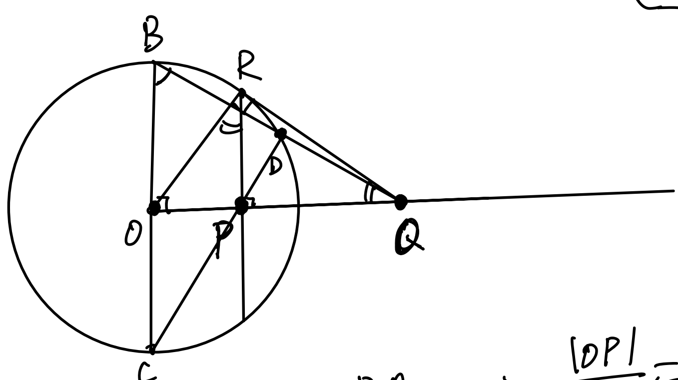


hyperbolic : ✓ ✓ ✓



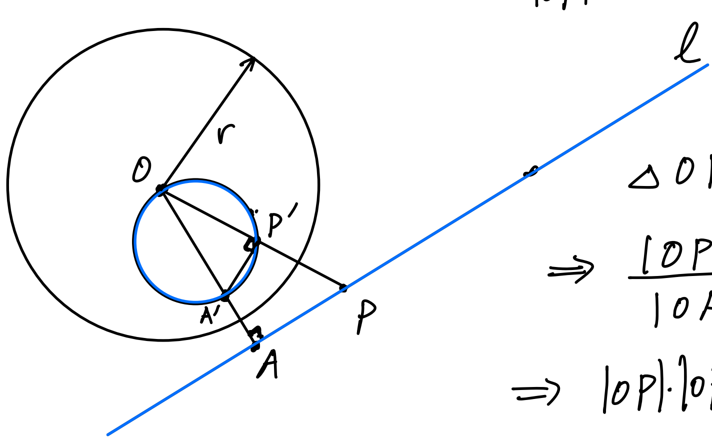


$$\boxed{|AP| \cdot |AQ| = r^2.}$$



$\triangle OPR \sim \triangle ORQ \Rightarrow \dfrac{|OP|}{|OR|} = \dfrac{|OR|}{|OQ|} \Rightarrow \underline{|OP| \cdot |OQ| = |OR|^2}.$

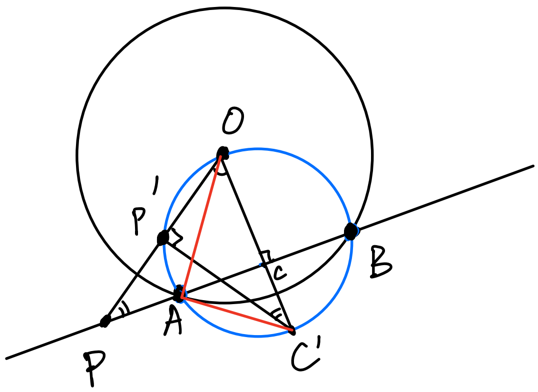
$\triangle OBQ \sim \triangle OPC \Rightarrow \dfrac{|OB|}{|OP|} = \dfrac{|OQ|}{|OC|} \Rightarrow |OP| \cdot |OQ| = \underbrace{|OB| \cdot |OC|}_{= r^2}$



$\triangle OP'A' \sim \triangle OAP$

$\Rightarrow \dfrac{|OP'|}{|OA|} = \dfrac{|OA'|}{|OP|}$

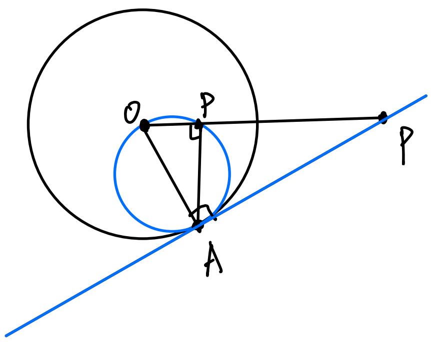
$\Rightarrow |OP| \cdot |OP'| = |OA| \cdot |OA'| = r^2$



$$\frac{|OP'|}{|OC|} = \frac{|OC'|}{|OP|} \Rightarrow |OP'|\cdot|OP| = |OC|\cdot|OC'| = r^2$$

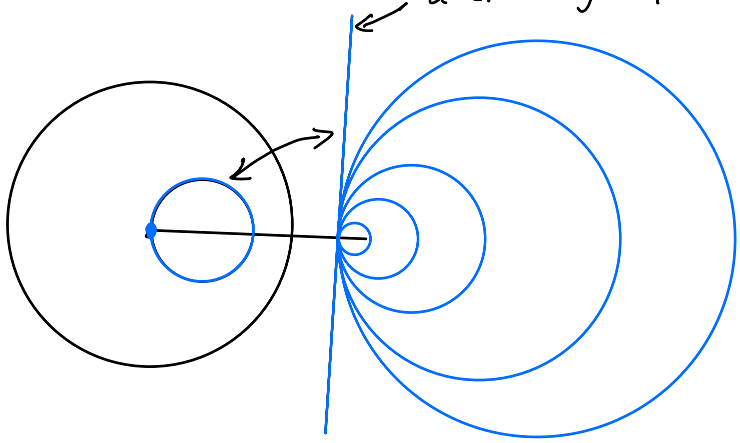
$$\Uparrow$$

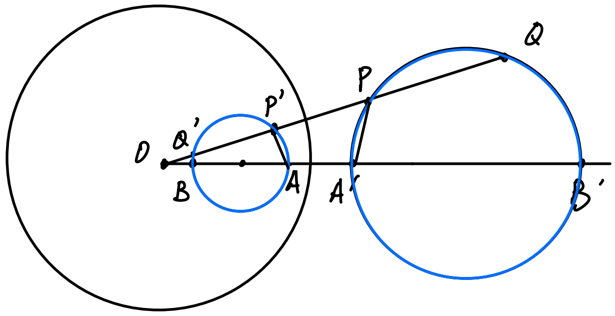
$$\triangle OC'P' \sim \triangle OPC$$



$$|OP|\cdot|OP'| = |OA|^2 = r^2.$$

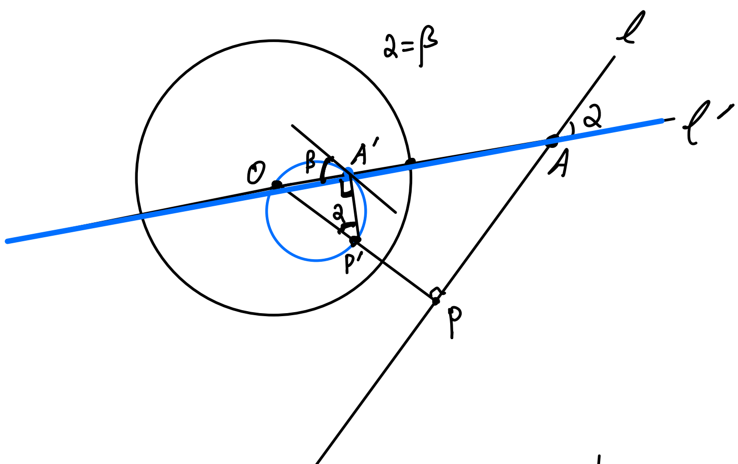
a circle of infinite radius.

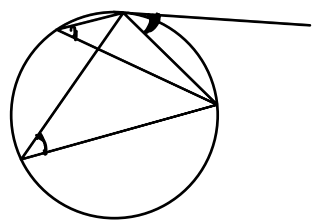


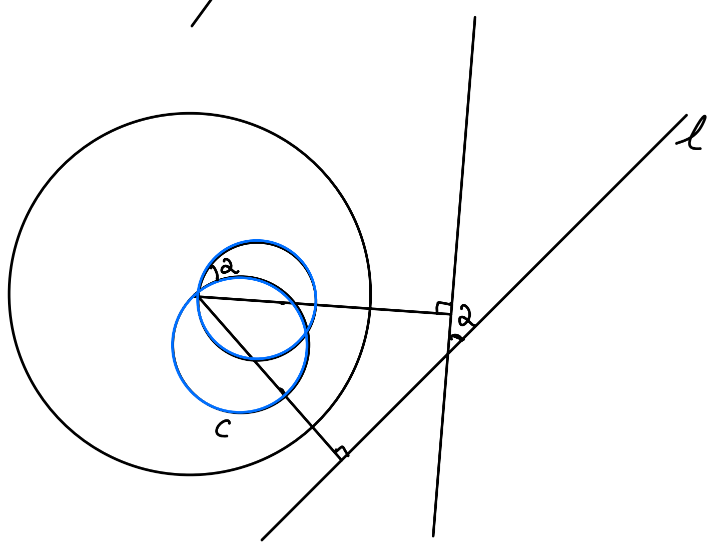


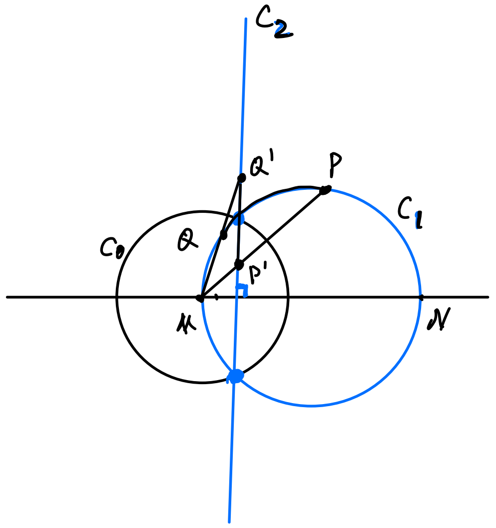
1. $\boxed{\text{Inversion maps circles to circles. (may be of infinite radius).}}$

2. Inversion preserves angles.









geodesic = "shortest curves"

reflection = "inversion"