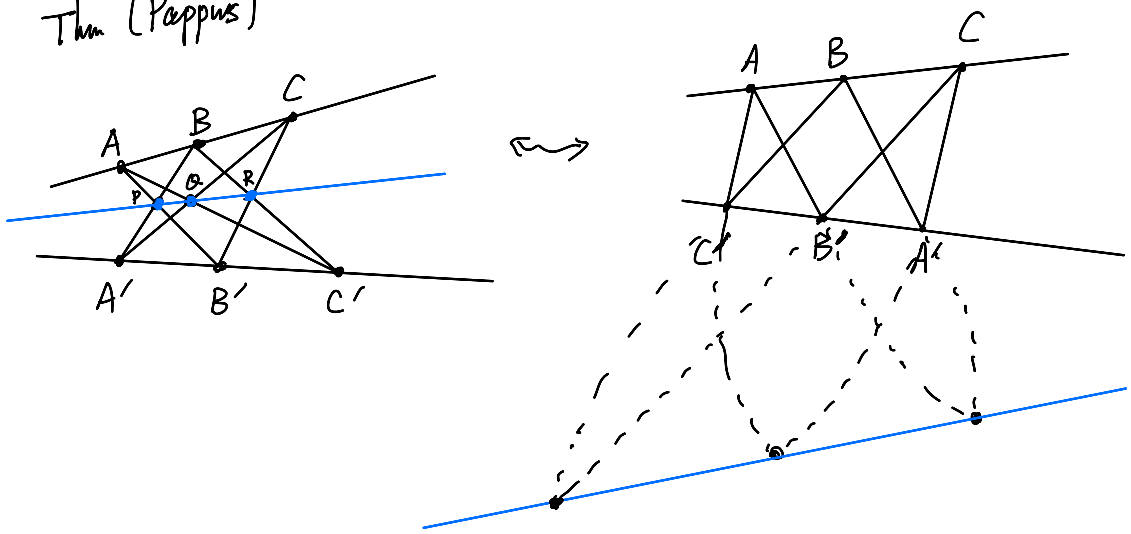
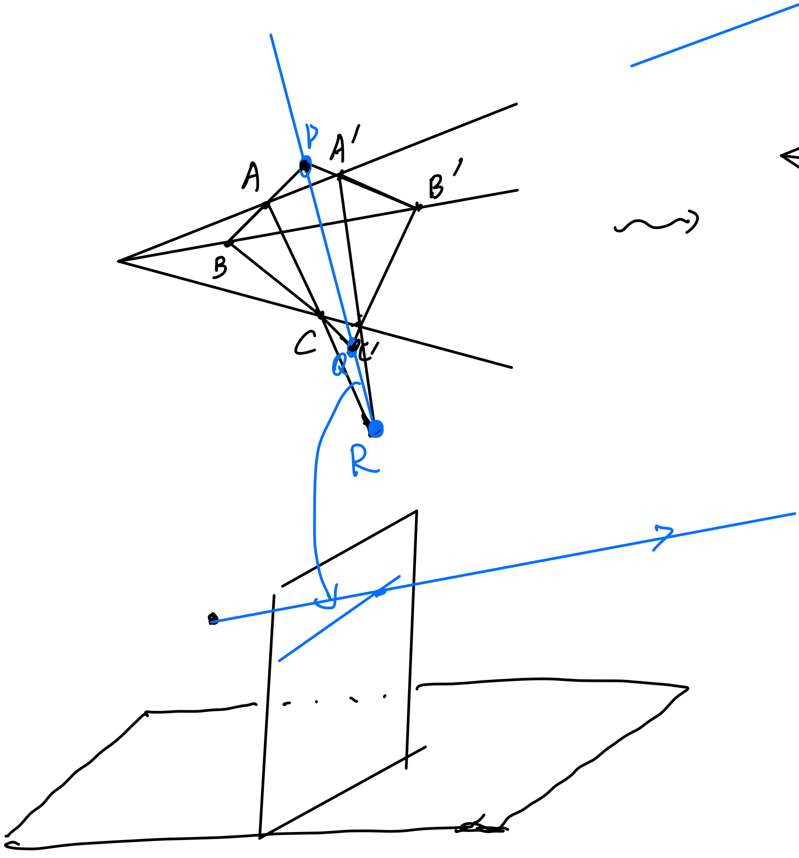
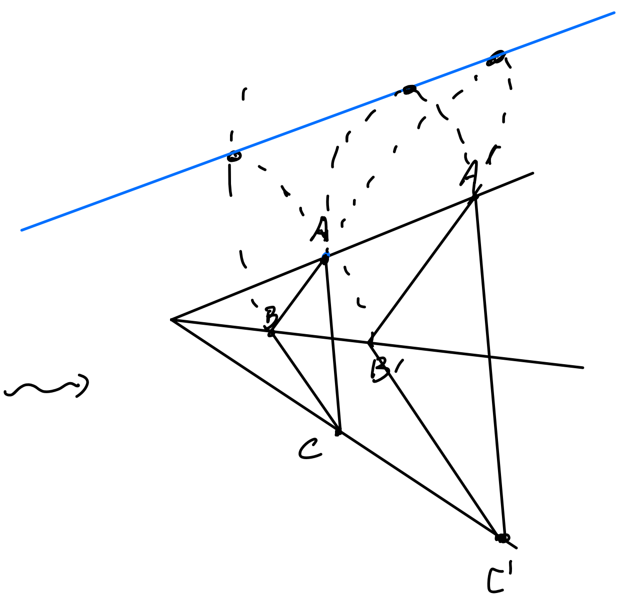
Projective Geometry.

Thm (Pappus)

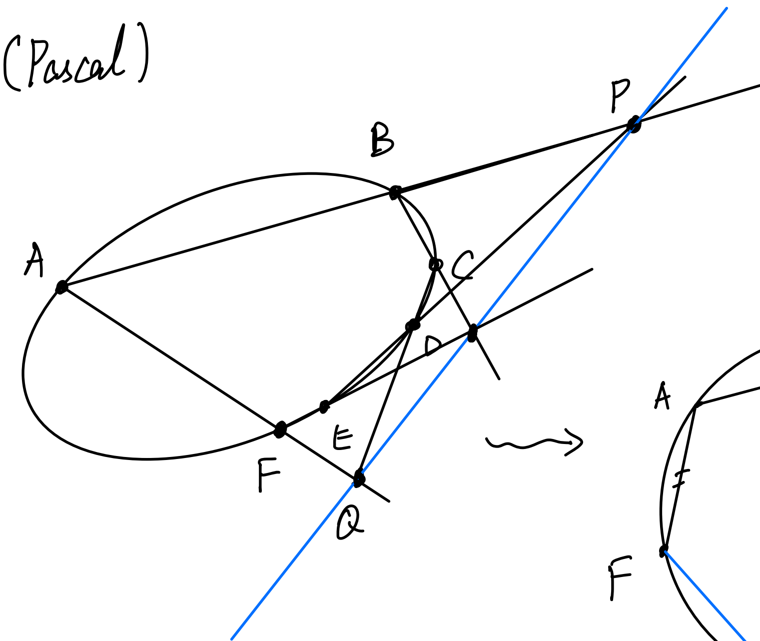


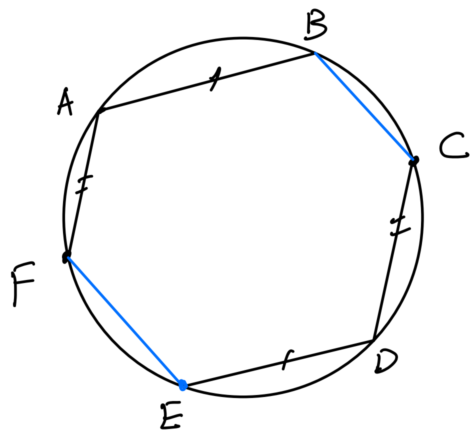
Thm (Desargues)



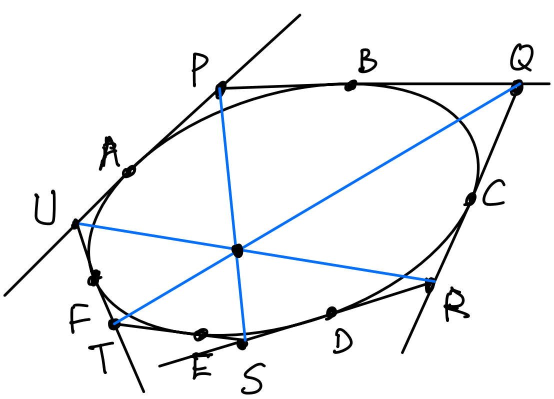


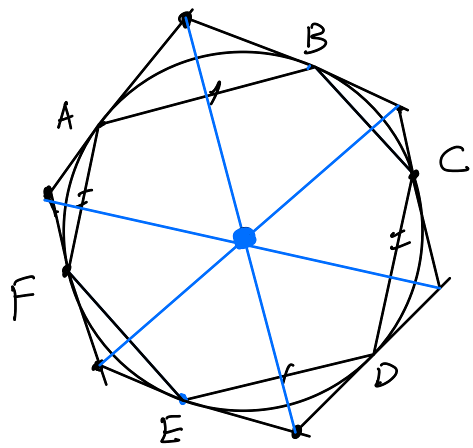
Thm (Pascal)

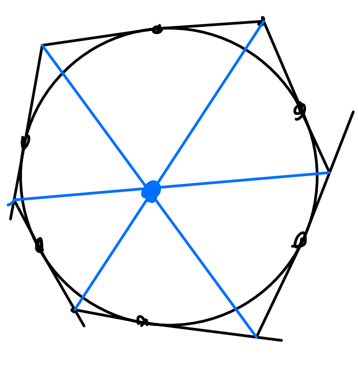


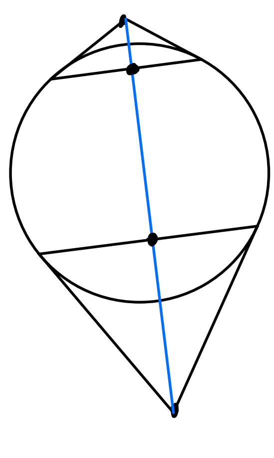


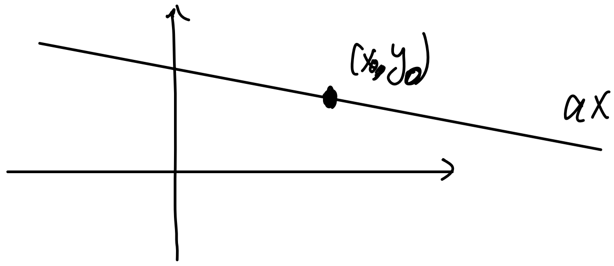
Thm (Brianchon)



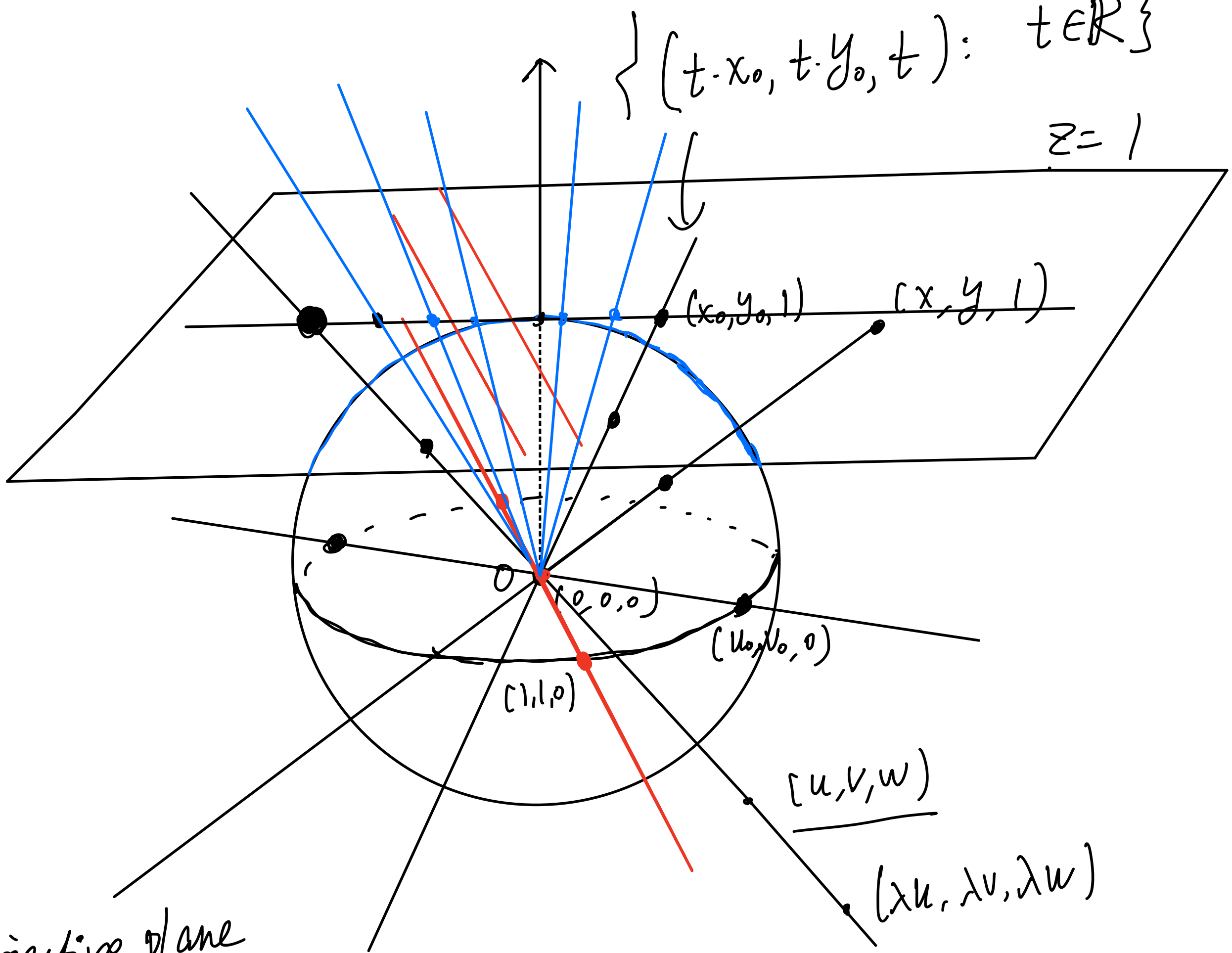








$\Longrightarrow$ $\boxed{ax_0+by_0=1}$



Projective plane

$$\mathbb{P}^2=\mathbb{R}^2\cup L_\infty$$

$$\begin{array}{l}\cup\!\!\!\!\!\; \\ P=[u,v,w]=[\lambda\cdot u,\lambda v,\lambda w]\end{array}$$

homogeneous coordinate

$(0,0,0)\neq[u,v,w]$

$[\frac{1}{2},\frac{1}{2},1] \leftsquigarrow [1,1,2] = [2,2,4] = [3,3,6] = [-1,-1,-2]$

$(\frac{1}{2},\frac{1}{2})$

$[1,1,0] = [2,2,0] = [-1,-1,0] = [100,100,0]$

$\{[x,y,1], x,y \in \mathbb{R}\}$ $\quad [x,y,1] = [\lambda x, \lambda y, \lambda]$

$$\mathbb{P}^2 = \mathbb{R}^2 \cup L_\infty = \{[x,y,0], x,y \in \mathbb{R}\}$$

$$P = [u,v,w] \underset{\lambda \neq 0}{\sim} [\lambda u, \lambda v, \lambda w]$$

$$[u,v,w] = \begin{cases} [\frac{u}{w},\frac{v}{w},\frac{w}{w}] = [\frac{u}{w},\frac{v}{w},1] = (\frac{u}{w},\frac{v}{w}) \in \mathbb{R}^2 & w \neq 0. \\ [u,v,0] & w = 0. \end{cases}$$

$(0,0,0) \neq (u,v,w)$

$[u,v,0]$: direction parallel to $(u,v) \neq (0,0)$

$\mathbb{R}^2 \ni (2,3) \rightsquigarrow [2,3,1] = [4,6,2] = [-2,-3,-1]$

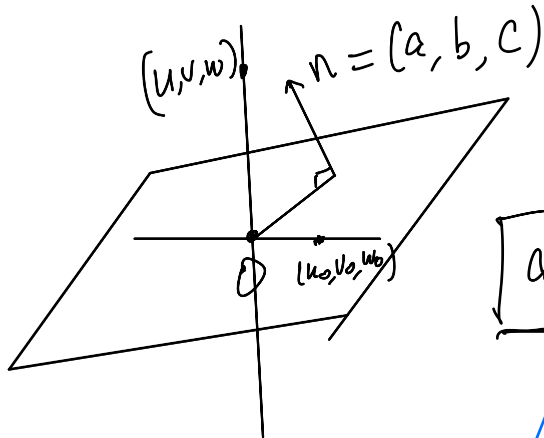
$L_\infty \ni$ direction parallel to $\overrightarrow{OP} = (2,3)$ $\rightsquigarrow [2,3,0] = [4,6,0] = [-2,-3,0]$.

Point in $\mathbb{P}^2$ $\rightsquigarrow$ line in $\mathbb{R}^3$ $\quad [u,v,w]$

$\boxed{au+bv+cw=0}$

Lines in $\mathbb{P}^2$ $\rightsquigarrow$ plane in $\mathbb{R}^3$ (a, b, c).

passing through O



$$a \cdot x + b \cdot y + c \cdot z = 0.$$

$$\boxed{a \cdot u_0 + b \cdot v_0 + c \cdot w_0 = 0.}$$