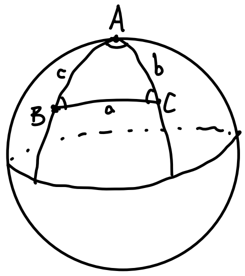


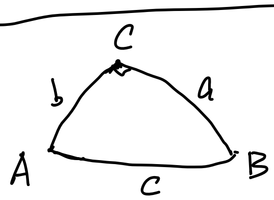
$|\triangle ABC| = \angle A + \angle B + \angle C - \pi$

$\cos c = \cos a \cdot \cos b + 2 \sin a \cdot \sin b \cdot \cos\angle C$

$\Updownarrow$

$\left(c^2 = a^2 + b^2 - 2a \cdot b \cos\angle C \right).$

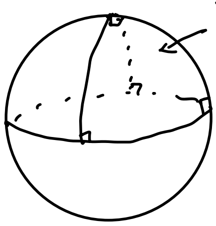
$$\frac{\sin a}{\sin\angle A} = \frac{\sin b}{\sin\angle B} = \frac{\sin c}{\sin\angle C}.$$



$\cos c = \cos a \cdot \cos b$

$\sin A = \frac{\sin a}{\sin c} \leftarrow a$, $\quad \cos A = \frac{\sin b}{\sin c} \cdot \cos a.$

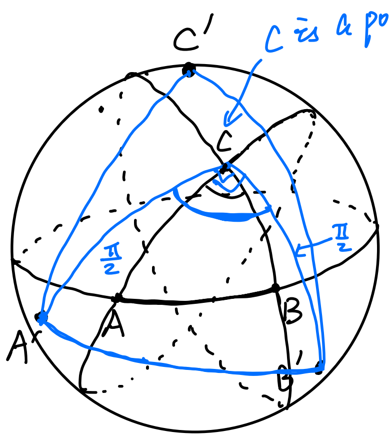
$\sin A \approx \frac{a}{c}$ $\qquad \cos A \approx \frac{b}{c}$ $\qquad 1 - \frac{a^2}{2} \approx 1$



$\frac{4\pi}{8} = \frac{\pi}{2} = \left(\frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} - \pi \right)$

Area of sphere $= 4\pi$

C is a pole w.r.t. $\widehat{A'B'}$



$\cos c = \cos a \cdot \cos b + 2 \sin a \cdot \sin b \cdot \cos\angle C$

$\Updownarrow$

$\boxed{\cos\angle C = -\cos\angle A \cdot \cos\angle B + 2 \sin\angle A \cdot \sin\angle B \cdot \cos c}$

$\angle C = \angle ACB = \pi - \angle A'CB' = \pi - c'$

$\angle B = \pi - b'$

$\angle A = \pi - a'$

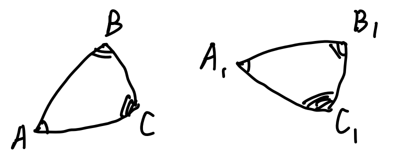
$\triangle ABC \xleftrightarrow{\text{polar}} \triangle A'B'C'.$

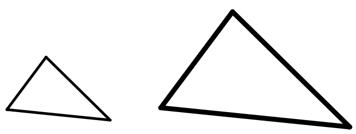
$$\cos(\pi-\angle C)=\cos(\pi-\angle A)\cdot\cos(\pi-\angle B)+2\cdot\sin(\pi-\angle A)\cdot\sin(\pi-\angle B)\cdot\cos(\pi-c).$$

$$\cos c' \qquad \cos a' \qquad \cos b' \qquad \sin a' \qquad \sin b' \qquad \cos\angle C'$$

$$+\cos\angle C=-\cos\angle A\cdot\cos\angle B+2\cdot\sin\angle A\cdot\sin\angle B\cdot\cos C.$$

Criterion for congruent spherical triangles: SSS $\updownarrow$ AAA, SAS $\updownarrow$ ASA





$\triangle ABC \qquad \triangle A_1B_1C_1$

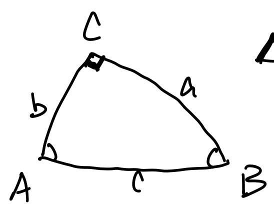
$\updownarrow \qquad \updownarrow$

$\triangle A'B'C' \qquad \triangle A_1'B_1'C_1'$

$c'=\pi-\angle A \qquad c_1'=\pi-\angle A_1'$

$b'=\pi-\angle B \qquad b_1'=\pi-\angle B_1'$

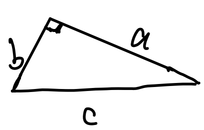
$c'=\pi-\angle C \qquad c'=\pi-\angle C_1'$



$$\Delta=|\triangle ABC|=\angle A+\angle B+\frac{\pi}{2}-\pi=\angle A+\angle B-\frac{\pi}{2}.$$

$$\sin(\alpha-\tfrac{\pi}{2})=-\sin(\tfrac{\pi}{2}-\alpha)=-\cos\alpha$$

$$\begin{aligned}\sin\Delta&=\sin(\angle A+\angle B-\tfrac{\pi}{2})\\&=-\cos(\angle A+\angle B)\\&=-\cos\angle A\cdot\cos\angle B+\sin\angle A\cdot\sin\angle B\\&=-\frac{\sin b}{\sin c}\cdot\cos a\cdot\frac{\sin a}{\sin c}\cdot\cos b+\frac{\sin a}{\sin c}\cdot\frac{\sin b}{\sin c}\\&=\frac{1}{\sin^2 c}\cdot\sin a\cdot\sin b\cdot(1-\underbrace{\cos a\cdot\cos b}_{=\cos c}).\\&=\frac{1-\cos c}{(1-\cos^2 c)}\cdot\sin a\cdot\sin b=\frac{\sin a\cdot\sin b}{1+\cos c}=\frac{\sin a\cdot\sin b}{1+\cos a\cdot\cos b}\end{aligned}$$



$\left(\frac{1}{2}ab\right)$

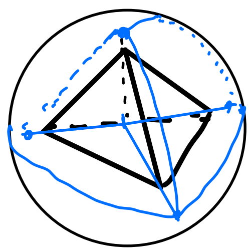
$$\sin\Delta = \frac{\sin a\cdot\sin b}{1+\cos a\cdot\cos b}$$

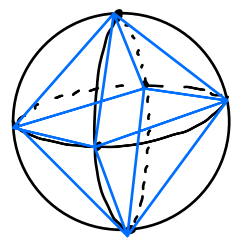
$$\wr$$

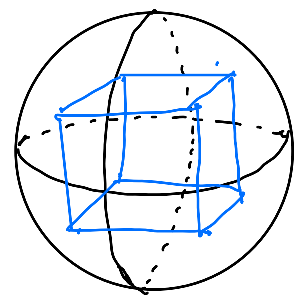
$$\Delta = \frac{a\cdot b}{1+\left(1-\frac{a^2}{2}\right)\left(1-\frac{b^2}{2}\right)} = \frac{a\cdot b}{2}$$

Tiling of sphere　　　　　　　　semiregular

regular solids:　tetrahedron　cube　octahedron　dedo.　Ico.



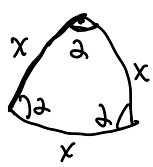




$$\frac{4\pi}{4} = \pi$$

$$\pi - \frac{\pi}{3} = 120^\circ$$

$$\shortparallel$$



$$3\alpha - \pi = \pi \Rightarrow \alpha = \frac{2\pi}{3}$$

$$\cos\alpha = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$\sin\alpha = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

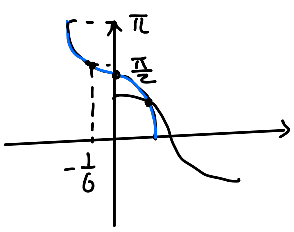
$$\frac{\sin x}{\sin\alpha}$$

$$\boxed{\cos\alpha = -\cos\alpha\cdot\cos\alpha + 2\cdot\sin\alpha\cdot\sin\alpha\cdot\cos x}$$

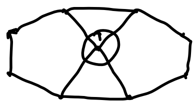
$$\Rightarrow\quad \cos x = \frac{\cos\alpha + \cos^2\alpha}{2\sin^2\alpha} = \frac{-\frac{1}{2}+\frac{1}{4}}{2\cdot\frac{3}{4}} = \frac{-\frac{1}{4}}{\frac{3}{2}} = -\frac{1}{6}$$

$$0 < x < \pi$$

$$\Rightarrow\quad x = \cos^{-1}\left(-\frac{1}{6}\right) = \pi - \cos^{-1}\left(\frac{1}{6}\right).$$

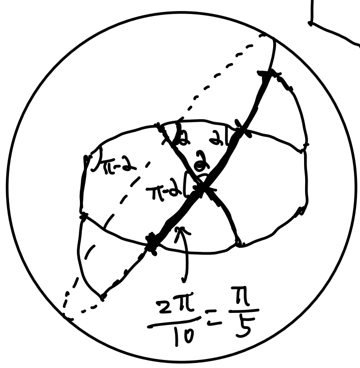


semiregular: $(3,5,3,5)$ icosidodecahedron



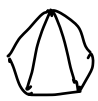
$\boxed{F_3 = 20}$, $\boxed{F_5 = 12}$

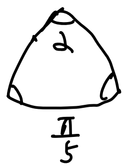
$60\times 2 + 108\times 2 = 168\times 2 = 336 < 360^\circ$



$$(3\alpha-\pi)\cdot 20 + \underbrace{(5(\pi-\alpha)-3\pi)}_{=\,(2\pi-5\alpha)}\cdot 12 = 4\pi$$

$$\cancel{60\alpha} - 20\pi + 24\pi - \cancel{60\alpha} = 4\pi.$$





$$\cos\frac{\pi}{5} = \cos\frac{\pi}{5}\cdot\cos\frac{\pi}{5} + 2\cdot\sin\frac{\pi}{5}\cdot\sin\frac{\pi}{5}\cdot\cos\alpha.$$

$$\Rightarrow \cos\alpha = \frac{\cos\frac{\pi}{5} - \cos^2\frac{\pi}{5}}{2\cdot\sin^2\frac{\pi}{5}} = \frac{1}{\sqrt{5}}$$

$$\cos\left(\frac{\pi}{5}\right) = \frac{1+\sqrt{5}}{4}$$

36°

$\Rightarrow$ Area of $\triangle = 3\alpha - \pi = 3\cdot\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) - \pi.$

1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	0
0°	30°	60°	45°	90°

36° △ □

$\frac{1+\sqrt{5}}{4}$ cos ⬠

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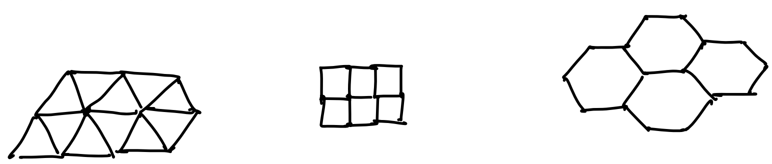
tiling on the plane



Semiregular tilling