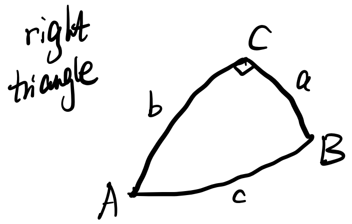
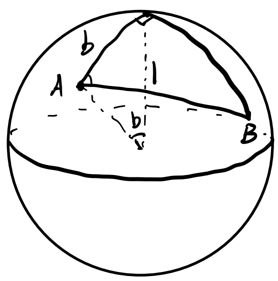
spherical trigonometry



Pythagorean Thm: $\underline{\cos c = \cos a \cdot \cos b}$

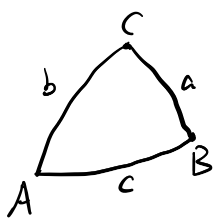
$\underset{\uparrow \text{ angle}}{\sin A} = \dfrac{\sin a}{\sin c}$, $\cos A = \dfrac{\sin b}{\sin c} \cos a$



a

$\wr$ a small

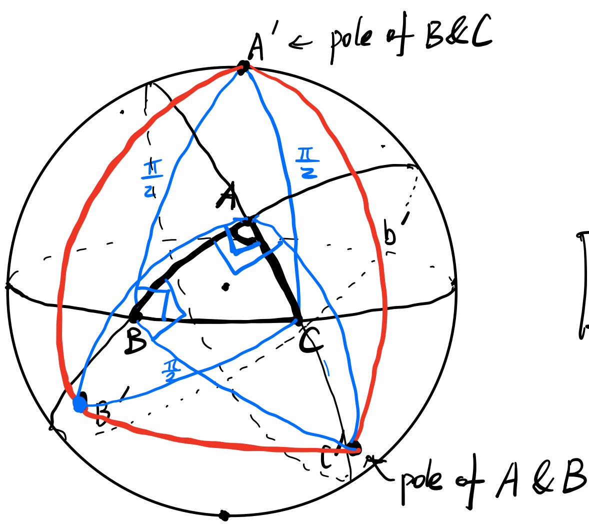
Law of Sine: $\dfrac{\sin a}{\sin A} = \dfrac{\sin b}{\sin B} = \dfrac{\sin c}{\sin C}$



Law of cosine: $\underset{\substack{\wr \\ 1-\frac{c^2}{2}}}{\cos c} = \underset{\substack{\wr \\ 1-\frac{a^2}{2}}}{\cos a} \cdot \underset{\substack{\wr \\ 1-\frac{b^2}{2}}}{\cos b} + \underset{\substack{\wr \\ a}}{\sin a} \cdot \underset{\substack{\wr \\ b}}{\sin b} \cdot \cos(\angle C)$

$\left(\rightsquigarrow \; c^2 = a^2 + b^2 - 2ab \cdot \cos C \right)$

Law of cosine for angles: $\cos\angle C = -\cos\angle A \cdot \cos\angle B + \sin\angle A \cdot \sin\angle B \cos(c)$

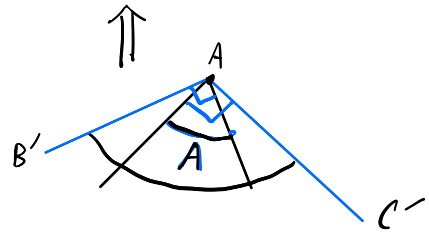


$\Delta ABC \leftrightsquigarrow \Delta A'B'C'$ polar triangle of ΔABC

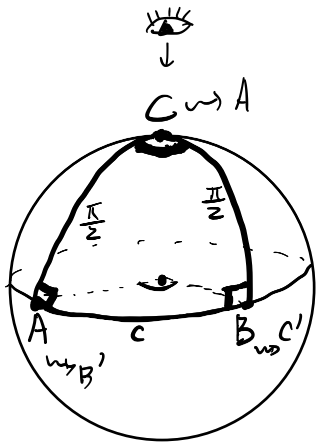
polar to each other

$$\boxed{\angle A = \pi - \angle B'AC' = \pi - a'}$$

$\Uparrow$



$\angle B'AC' + \angle A = 2 \cdot \frac{\pi}{2} = \pi$



$\underline{\angle C = c}$

$\angle A = \pi - \angle B'AC' = \pi - a'$ $\quad a' = \pi - \angle A$

$\angle B = \pi - \angle A'BC' = \pi - b'$ $\quad b' = \pi - \angle B$

$\underline{\angle C = \pi - \angle A'CB' = \pi - c'}$ $\quad c' = \pi - \angle C$

$\cos(\pi - \alpha) = -\cos\alpha$

$\sin(\pi - \alpha) = \sin\alpha$

$\angle A' = \pi - a$

$\angle B' = \pi - b$

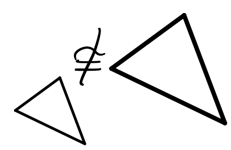
$\angle C' = \pi - c.$

$$\cos(c') = \underbrace{\cos(a')}_{=\cos(\pi - \angle A)} \cdot \underbrace{\cos(b')}_{=\cos(\pi - \angle B)} + \underbrace{\sin(a')}_{=\sin(\pi - \angle A)} \cdot \underbrace{\sin(b')}_{=\sin(\pi - \angle B)} \cdot \cos \underbrace{\angle C'}_{=\pi - c}$$

$$\cos(c') = \cos(\pi - \angle C)$$

$$-\cos\angle C = \cos\angle A \cdot \cos\angle B - \sin\angle A \cdot \sin\angle B \cdot \cos c$$

$(\angle C = \pi - (\angle A + \angle B) \text{ on plane})$

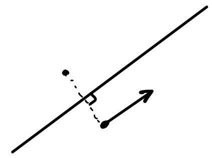


Congruence axiom on plane: SSS, SAS, ASA ~~AAA~~

axiom on sphere: SSS SAS ASA AAA

$$\text{AAA}\left\{\begin{array}{l} \triangle ABC \rightsquigarrow \triangle A'B'C' \\ \triangle DEF \rightsquigarrow \triangle D'E'F' \end{array}\right\} \text{SSS} \Rightarrow \triangle A'B'C' \cong \triangle D'E'F' \Downarrow \triangle ABC \cong \triangle DEF$$

Isometry on planes. translation / rotation / reflection / glide reflection



1.15

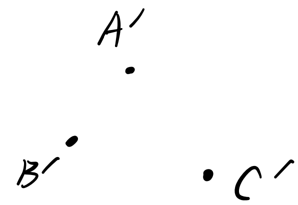
1.32: Two isometries f, g. $\triangle ABC$ (non-degenerate) triangle.

$$\left.\begin{array}{l} A' = f(A) = g(A) \\ B' = f(B) = g(B) \\ C' = f(C) = g(C) \end{array}\right\} \Rightarrow \boxed{f(P) = g(P) \quad \forall P}$$

Pf: $\underline{d(f(P), A')} = d(f(P), f(A)) = d(P, A) = d(g(P), g(A))$ $= \underline{d(g(P), A')}$

$d(f(P), B') = d(g(P), B')$

$d(f(P), C') = d(g(P), C')$

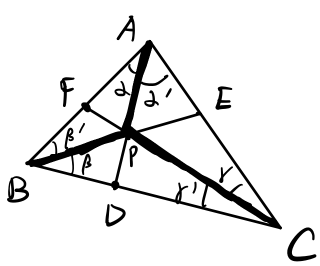


$\Rightarrow \underline{f(P) = g(P).}$

(If not ($f(P) \neq g(P)$), then $\underline{A', B', C'}$ are contained in the equidistant line between $f(P)$ and $g(P)$. Contradiction.)

- Ceva Thm $\overline{AD}, \overline{BE}, \overline{CF}$ concurrent (pass through a common pt.)

$\Updownarrow$



$$\underbrace{\frac{|AF|}{|FB|}}_{\parallel} \cdot \underbrace{\frac{|BD|}{|DC|}}_{\parallel} \cdot \underbrace{\frac{|CE|}{|EA|}}_{\parallel} = 1. \quad (*)$$

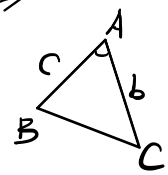
$$\frac{|\triangle ACP|}{|\triangle \cancel{BCP}|} \cdot \frac{\cancel{|\triangle ABP|}}{\cancel{|\triangle ACP|}} \cdot \frac{|\triangle BPC|}{\cancel{|\triangle ABP|}}$$

1.164 $\quad (*) \iff \sin\alpha \cdot \sin\beta \cdot \sin\gamma = \sin\alpha' \cdot \sin\beta' \cdot \sin\gamma'$.

$$\left(\frac{1}{2}|AB|\cdot|AP|\right)\left(\frac{1}{2}|BP|\cdot|BC|\right)\left(\frac{1}{2}|PC|\cdot|AC|\right)$$

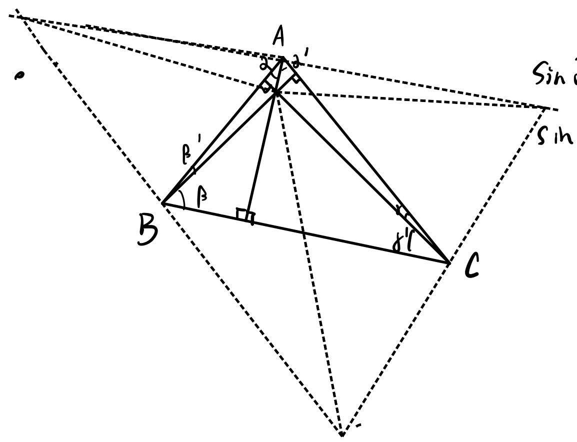
$$\parallel$$

$$|\triangle ABP|\cdot|\triangle BPC|\cdot|\triangle APC|$$



$|\triangle ABC| = \frac{1}{2} bc \cdot \sin A$.

1.168



$$\sin\alpha = \sin\left(\frac{\pi}{2} - \angle B\right) = \cos\angle B$$

$$\sin\alpha' = \sin\left(\frac{\pi}{2} - \angle C\right) = \cos\angle C$$

$$\sin\alpha\cdot\sin\beta\cdot\sin\gamma = \sin\alpha'\cdot\sin\beta'\cdot\sin\gamma'$$

$$\parallel \qquad\qquad\qquad \parallel$$

$$\cos\angle B\cdot\cos\angle C\cdot\cos\angle A = \cos\angle C\cdot\cos\angle A\,\cos\angle B$$