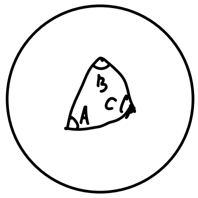
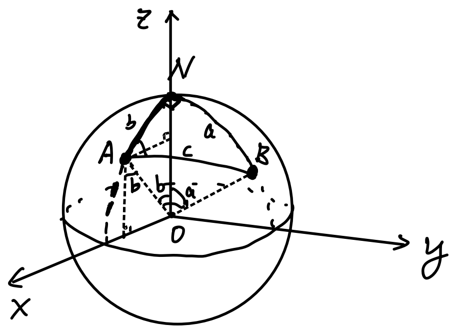
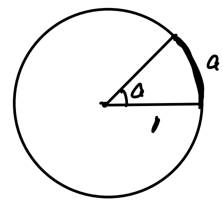


$|\triangle ABC| = (A+B+C-\pi)$ (when radius $= 1$)

spherical Pythagorean Thm : $\underline{\cos C = \cos a \cdot \cos b}$





$C = \angle AOB$

$\vec{OA} = A = (\sin b, 0, \cos b)$

$\vec{OB} = B = (0, \sin a, \cos a)$

$$\left(\begin{aligned} \cos a &= 1 - \frac{a^2}{2} + \underbrace{\frac{a^4}{4!} - \frac{a^6}{6!} + \cdots}_{} \\ &= 1 - \frac{a^2}{2} + \varepsilon \end{aligned}\right)$$

$\vec{OA} \cdot \vec{OB} = |\vec{OA}| \cdot |\vec{OB}| \cdot \cos \angle AOB = \cos C$

$\parallel$

$\cos a \cdot \cos b = \cos C$

$\parallel$ $\qquad\qquad \parallel$

$\underline{(1 - \frac{a^2}{2} + \varepsilon) \cdot (1 - \frac{b^2}{2} + \varepsilon)}$ $\qquad (1 - \frac{c^2}{2} + \varepsilon)$ $\qquad \propto |\varepsilon| << 1$

$\parallel$

$1 - \underline{\frac{1}{2}(a^2 + b^2)} + \varepsilon$ $\quad // \quad \underset{\varepsilon \to 0}{\Longrightarrow} \quad a^2 + b^2 = c^2$

$\angle NAB =$ angle between $H(NAO) =$ xz-plane and $H(NAB)$

$\qquad \Downarrow \qquad\qquad\qquad \Downarrow$

$\underline{\vec{n}_1 = (0, -1, 0)}$ $\qquad \vec{n}_2 = \vec{OA} \times \vec{OB}$

$\|\vec{n}_1\| = 1$

$$\vec{OA} \times \vec{OB} = \begin{vmatrix} i & j & k \\ \sin b & 0 & \cos b \\ 0 & \sin a & \cos a \end{vmatrix} = i \cdot (-\sin a \cdot \cos b) + (-j)(\sin b \cos a) + k \cdot (\sin b \sin a).$$

$$= \underline{(-\sin a \cdot \cos b, -\sin b \cos a, \sin b \sin a) = \vec{n}_2}$$

$$\|\vec{n}_2\|^2 = \underset{\substack{\parallel \\ 1-\cos^2 a}}{\sin^2 a}\,\cos^2 b + \underbrace{\sin^2 b \cdot \cos^2 a + \sin^2 b \cdot \sin^2 a}_{\sin^2 b} = \underline{1 - \cos^2 a \cdot \cos^2 b} = 1 - \cos^2 C = \sin^2 C$$

$\vec{n}_1 \cdot \vec{n}_2 = \|\vec{n}_1\| \cdot \|\vec{n}_2\| \cos A$

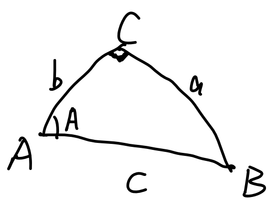
$= 1 \cdot \sin C \cdot \cos A = \sin b \cdot \cos a \implies \cos A = \dfrac{\sin b \cos a}{\sin C}$

$$\|\vec{n}_1 \times \vec{n}_2\| = \underline{\|\vec{n}_1\|\|\vec{n}_2\| \sin A = 1 \cdot \sin c \cdot \sin A} = \sin a \Longrightarrow \underline{\sin A = \frac{\sin a}{\sin c}}$$

$$\| \quad \left|\begin{array}{ccc} i & j & k \\ 0 & -1 & 0 \\ -\sin a \cos b & -\sin b \cos a & \sin b \sin a \end{array}\right| \begin{array}{l} \\ \leftarrow n_1 \\ \leftarrow n_2 \end{array} = \| i \cdot (-\sin b \cdot \sin a) - j \cdot 0 + k \cdot (-\sin a \cdot \cos b) \|$$

$$\sin^2 b \cdot \sin^2 a + \sin^2 a \cdot \cos^2 b = \sin^2 a$$

$$1 - \frac{a^2}{2!} + \frac{a^4}{4!} - \cdots$$

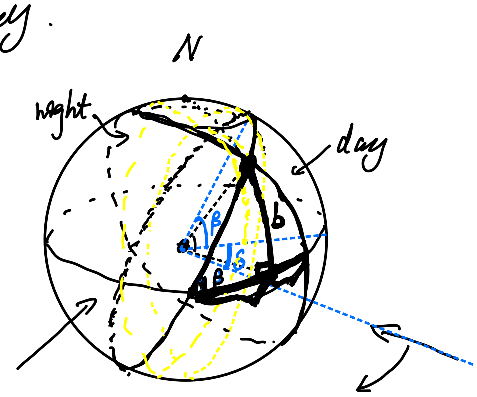


$$\cos A = \frac{\sin b \cdot \cos a}{\sin C} \qquad \sin A = \frac{\sin a}{\sin C} = \frac{a \cdot (1-o(1))}{c \cdot (1-o(1))} \to \frac{a}{C}$$

$$= \frac{b \cdot (1-o(1))\,(1-o(1))}{c \cdot (1-o(1))} \to \frac{b}{C} \gg \frac{b \cdot a^2}{c}$$

$$\sin b = b - \frac{b^3}{3!} + \frac{b^5}{5!} - \cdots = b \cdot (1 - \varepsilon)$$

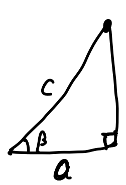
Wrocław in Poland Shortest day.



Latitude of city $= 51°07' = b$

Latitude of Sun $= -23°27' = \delta$

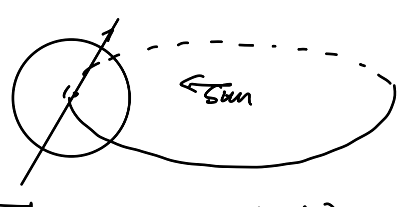
$\beta = 90° - 23°27' = 66°33'$



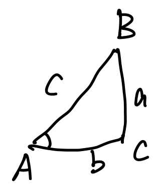
$$\sin\beta = \frac{\sin b}{\sin C}$$

$$\cos\beta = \frac{\sin a}{\sin C} \cdot \cos b$$

$$\Downarrow$$



$$\left(\tan\beta = \frac{b}{a}\right) \qquad \tan\beta = \tan b \cdot \frac{1}{\sin a} \Rightarrow \boxed{\sin a = \frac{\tan b}{\tan\beta}} \qquad a = \sin^{-1}\left(\frac{\tan b}{\tan\beta}\right) = 32.54°.$$



$$\sin A = \frac{\sin a}{\sin c}$$

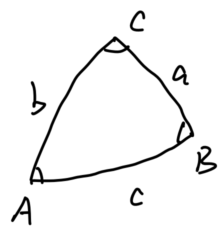
$$\cos A = \frac{\cos a \cdot \sin b}{\sin c}$$

hours

$$\left(12h - 12h \times \frac{32.54^\circ}{180^\circ} \times 2\right) = 7h\ 39^\circ \text{min}$$

length of the day.

Law of sine:



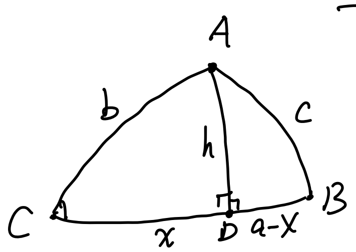
$$a \cdot (1 - o(1)) = \sin a$$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$\Downarrow$ a, b, c small $\to 0$

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin A} = \frac{c}{\sin C}}$$

Law of cosine:



$$\cos C = \frac{\sin x}{\sin b} \cdot \cos h$$

$$\cos c = \cos h \cdot \cos(a-x) = \cos h \cdot (\cos a \cos x + \sin a \cdot \sin x)$$

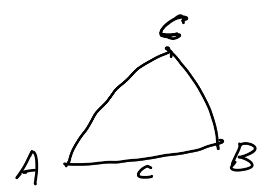
$$= \cos a \cdot \underbrace{(\cos h \cdot \cos x)}_{\cos b} + \sin a \cdot \underbrace{(\cos h \cdot \sin x)}_{\sin b \cdot \cos C}$$

$$\cos c = \cos a \cdot \cos b + \sin a \cdot \sin b \cdot \cos C$$

$$\cos c = 1 - \frac{c^2}{2}, \quad \cos a \cdot \cos b = \left(1 - \frac{a^2}{2}\right)\cdot\left(1 - \frac{b^2}{2}\right) = 1 - \frac{a^2+b^2}{2} + \frac{a^2b^2}{2}, \quad \sin a \cdot \sin b \cdot \cos C = a \cdot b \cdot \cos C$$

$$-\frac{c^2}{2} = -\frac{a^2+b^2}{2} + a \cdot b \cdot \cos C$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C.$$



Law of Cosine for sides: $\cos c = \cos a \cdot \cos b + \sin a \cdot \sin b \cos C$

$\Updownarrow$

Law of Cosine for angles: $\cos C = -\cos A \cdot \cos B + \sin A \cdot \sin B \cdot \cos c$