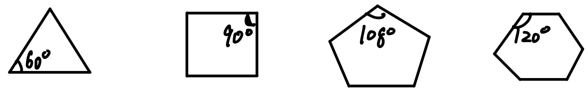
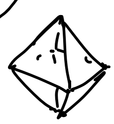


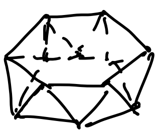
$\frac{(5-2)\times 180}{5}$

of faces at a vertex
≥ 3

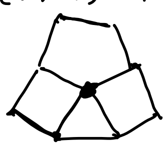
$\# = 5$
- $\#\triangle = 5$: Icosahedron
- $\#\triangle = 4 \rightsquigarrow 240°$
  - $\#\square = 1 \rightsquigarrow$ snub cube
  - $\#⬠ = 1 \rightsquigarrow$ snub dodecahedron
- $\#\triangle = 3 : \rightsquigarrow 180° + 90°\cdot 2 = 360°$ X
- ≤ 3 X

$\# = 4$
- $\#\triangle = 1$: $(60+90+108+120 = 258+120 = 378 > 360 X)$
  - $\#\square \geq 2$
    - $\#\square = 3$ → (3, 4, 4, 4)
    - $\#\square = 2$, $\#⬠$
  - $\#⬠ \geq 2$ X (3, 4, 5, 4)
    - $\left(\frac{108\times 2 + 90 + 60}{216+150 > 360}\right)$
  - $60 + 90\times 2 + 120 = 360$ X
- $\#\triangle = 2$
  - $\#\square = 2$ (3, 4, 3, 4)
  - $\#⬠ = 2$ (3, 5, 3, 5)
- $\#\triangle = 3$: drum (3, 3, 3, n)
- $\#\triangle = 4$: Octahedron

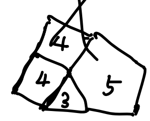


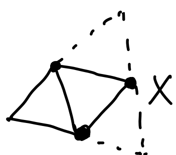


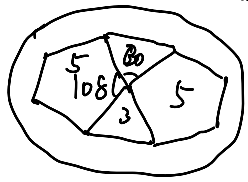
(3, 4, 5, 4)

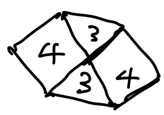


((3, 4, 4, 5) (3, 5, 4, 4))

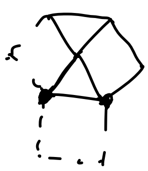


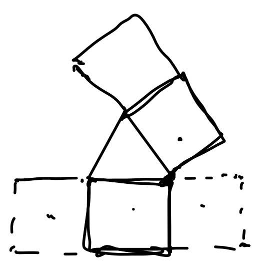






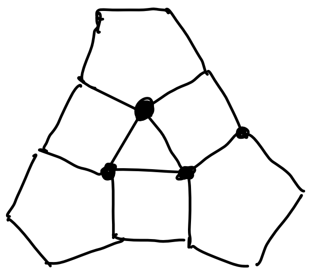
$(60+90)\times 2 = 150\times 2 = 300$





= 3 :

$(\underset{F_3}{3}, \underset{F_4}{4}, \overset{F_5}{5}, \underset{F_4}{4})$:



$$\begin{cases} \chi(S^2) = V - E + F = 2 \\ E = \dfrac{3F_3 + 4F_4 + 5F_5}{2} \\ F = F_3 + F_4 + F_5 \\ V = 3F_3 = \underbrace{\dfrac{4F_4}{2}}_{2F_4} = \dfrac{5F_5}{1} \end{cases}$$

$$\Downarrow$$

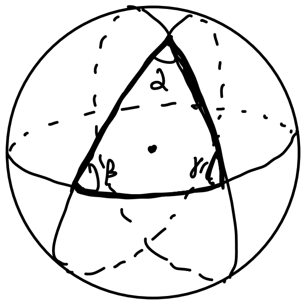
$F_4 = \frac{3F_3}{2}$, $F_5 = \frac{3F_3}{5}$.

$E = \frac{3F_3 + 6F_3 + 3F_3}{2} = 6F_3$, $F = F_3 + \frac{3F_3}{2} + \frac{3F_3}{5} = \frac{10+15+6}{10}F_3 = \frac{31}{10}F_3$

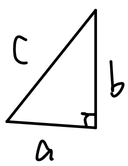
$2 = \underbrace{3F_3 - 6F_3}_{-3F_3} + \frac{31}{10}F_3 = \frac{31-30}{10}F_3 = \frac{1}{10}F_3 \Rightarrow F_3 = 20$

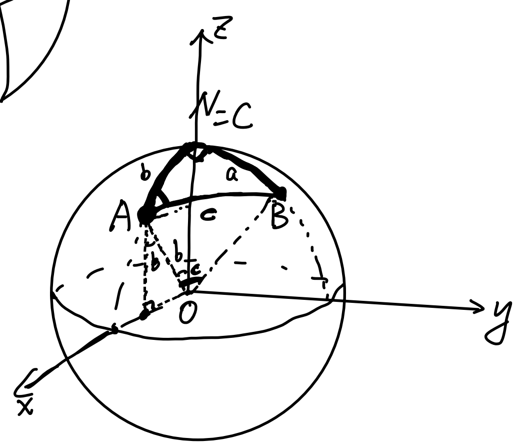
$F_4 = \frac{3\cdot 20}{2} = 30$, $F_5 = \frac{3\cdot 20}{5} = 12$.

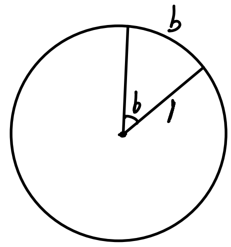
rhombicosidodecahedron



$$|\triangle| = \alpha+\beta+\gamma-\pi > 0$$

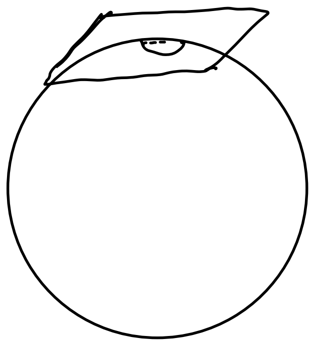






$A=(\sin b, 0, \cos b)$, $B=(0, \sin a, \cos a)$

$(\vec{v}\cdot\vec{w} = |\vec{v}||\vec{w}|\cos\theta = \cos\theta)$ $A\cdot B = \boxed{\cos b\cdot\cos a = \cos C}$



spherical pythagorean Thm.

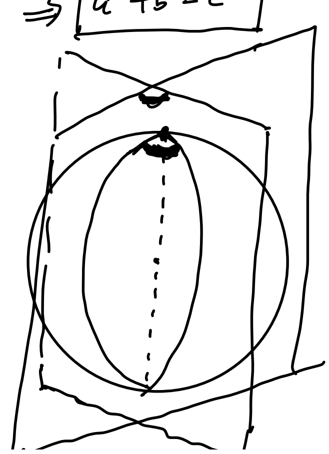
$$\cos(x) = 1-\frac{x^2}{2}+\left(\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots\right) = 1-\frac{x^2}{2}+\underbrace{o(|x|^2)}_{\text{small error term}}$$

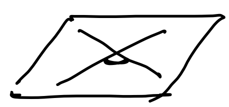
$$\cos a\cdot\cos b = \cos C = 1-\frac{c^2}{2}+\varepsilon$$

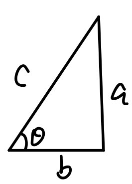
$$= \left(1-\frac{a^2}{2}+\varepsilon\right)\cdot\left(1-\frac{b^2}{2}+\varepsilon\right)$$

$$= 1-\frac{a^2}{2}-\frac{b^2}{2}+\varepsilon$$

$$\Rightarrow \boxed{a^2+b^2=c^2}$$

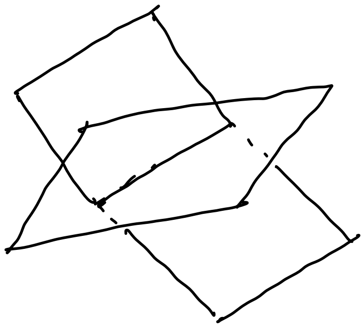


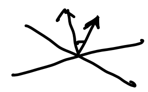




$\sin\theta = \frac{a}{c}$.

$\cos\theta = \frac{b}{c}$.





xz-plane: $\vec{n}_1 = (0,-1,0)$.

$$\vec{n}_2 = A \times B = \begin{vmatrix} i & j & k \\ \sin b & 0 & \cos b \\ 0 & \sin a & \cos a \end{vmatrix} = i \cdot (-\sin a \cdot \cos b) - j \cdot (\sin b \cdot \cos a) + k \cdot (\sin b \sin a)$$

$$\cos A = \frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \cdot \|\vec{n}_2\|} = \frac{+\sin b \cdot \cos a}{1 \cdot \sqrt{\underbrace{\sin^2 a \cdot \cos^2 b}_{=(1-\cos^2 a)\cdot\cos^2 b} + \underbrace{\sin^2 b \cdot \cos^2 a + \sin^2 b \sin^2 a}_{=\sin^2 b}}} = \frac{\sin b \cdot \cos a}{\sin C}$$

$$(1-\cos^2 a)\cdot\cos^2 b + \sin^2 b = 1 - \cos^2 C = \sin^2 C$$

$$\boxed{\cos C = \cos a \cdot \cos b}$$

$$\boxed{\cos A = \frac{\sin b \cdot \cos a}{\sin C} \qquad \sin A = \frac{\sin a}{\sin C}}$$