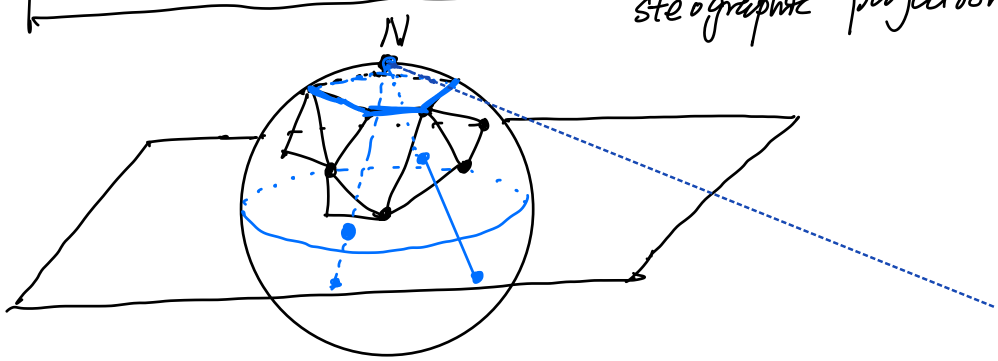
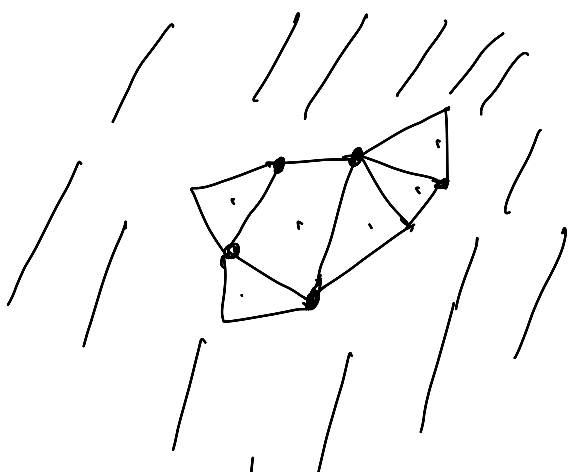
regular solids ⟷ platonic solids.

	tetrahedron	cube ⟷	octahedron	dodecahedron ⟷	isosahedron
V	4	8	6	20	12
E	6	12	12	30	30
F	4	6	8	12	20.

$$\boxed{\chi(S^2) = V - E + F = 2}$$

steographic projection





$v = V$

$e = E$

$f = F - 1$

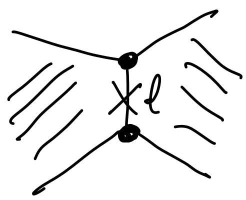
$$v - e + f = \underbrace{V - E + F}_{2} - 1 = 1$$

Thm: Let R be a finite connected (filled) planar graph

Then $\boxed{\chi(R) = v - e + f = 1}$ $(\Rightarrow \chi(S^2) = 2)$

Pf: Induction on the number of edges.

- $e=0$. $v=1$, $f=0$. $v-e+f=1$
- Assume $e \le k$ ✓. Consider $e=k+1$



$R' = R \setminus \{\ell\}$

Case 1: R' is connected.

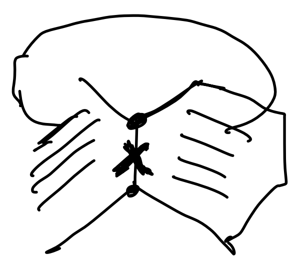
$v'=v$, $e'=\overset{k}{\overset{\shortparallel}{e}}-1$, $f'=f-1$

$v'-e'+f'=v-e+f=1$.

Case 2: R' is disconnected. $R'=R_1' \cup R_2'$

$\chi(R_1')=\chi(R_2')=1$.

$\chi(R')=\chi(R_1')+\chi(R_2')=2$.

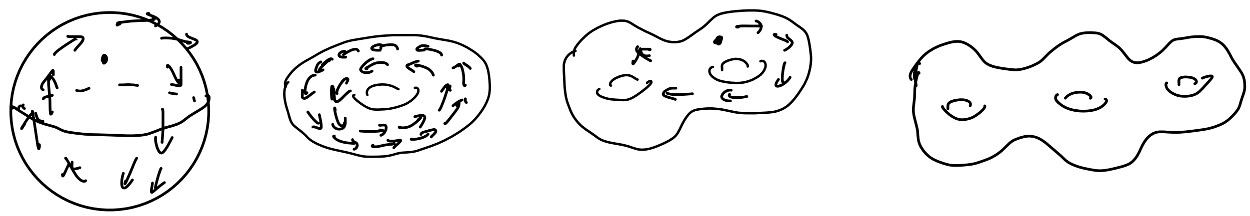


$v'=v$, $e'=e-1$, $f'=f$

$v'-e'+f'=v-e+1+f=2$

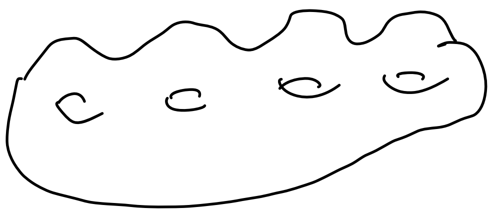
$\Rightarrow \chi(R)=v-e+f=1$.

∎



$\chi = 2 \qquad 0 \qquad -2 \qquad -4$

g-handles



$\Rightarrow \underline{\chi(\Sigma_g) = 2-2g.}$

$$\sum_{p\in \mathrm{zero}(V)} \mathrm{ind}_p(\vec{v}) = \chi(\Sigma_g) = 2-2g.$$

(Hopf Thm)

semiregular polyhedra $\xleftrightarrow{13}$ Archimedean solids.

truncated (vertex)
- Te
- Cu, Oc > cuboctahedron
- Do, Ic > icosidodecahedron

truncated edge
- rhombicuboctahedron
- rhombicosidodecahedron

truncated cuboctahedron

truncated icosi...

snub cube

snub dodecahedron.

Ex: Snub dodecahedron. $(3,3,3,3,5)$

$F_3 = \#$ 3-gon. $\quad F_5 = \#$ 5-gon.

$$V = 5F_5 = \frac{3F_3}{4}, \quad E = \frac{3F_3 + 5F_5}{2}, \quad F = F_3 + F_5$$

$$E = \frac{25F_5}{2}, \quad F = \frac{20}{3}F_5 + F_5$$

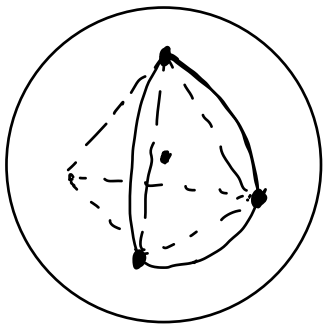
$$\chi(S^2) = 2 = V - E + F = 5F_5 - \frac{25}{2}F_5 + \frac{23}{3}F_5 = \frac{F_5}{6}(30 - 75 + 46) = \frac{F_5}{6}$$

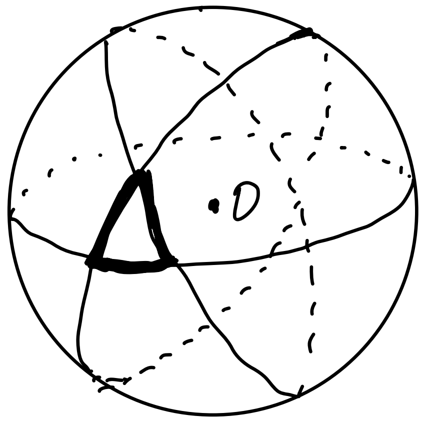
$$\Rightarrow F_5 = 2 \cdot 6 = 12$$

$$F_3 = \frac{20F_5}{3} = 80$$

$$\Rightarrow \quad V = 60, \quad E = \frac{25 \cdot 12}{2} = 150, \quad F = 92.$$

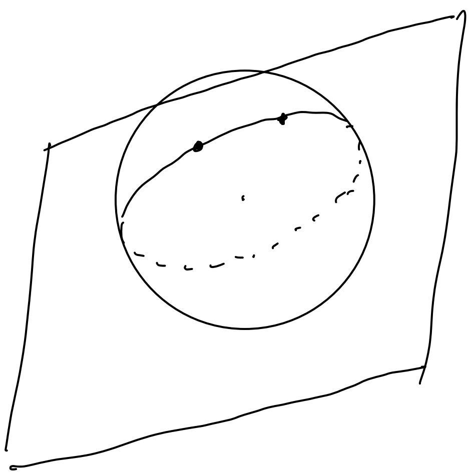
Similar calculation for snub cub $(3,3,3,3,4)$

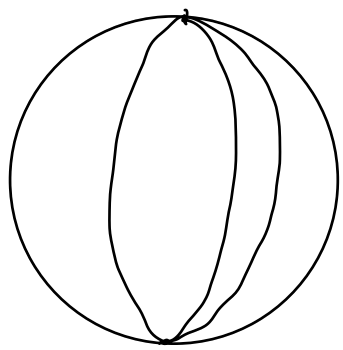


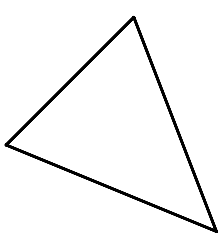


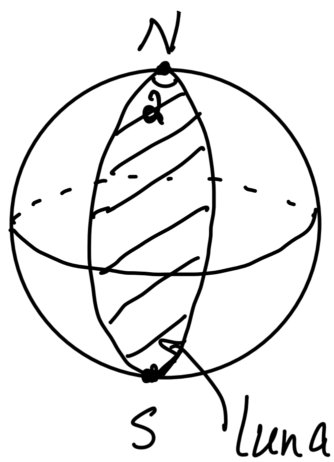
$r = 1$

$4\pi r^2 = 4\pi$



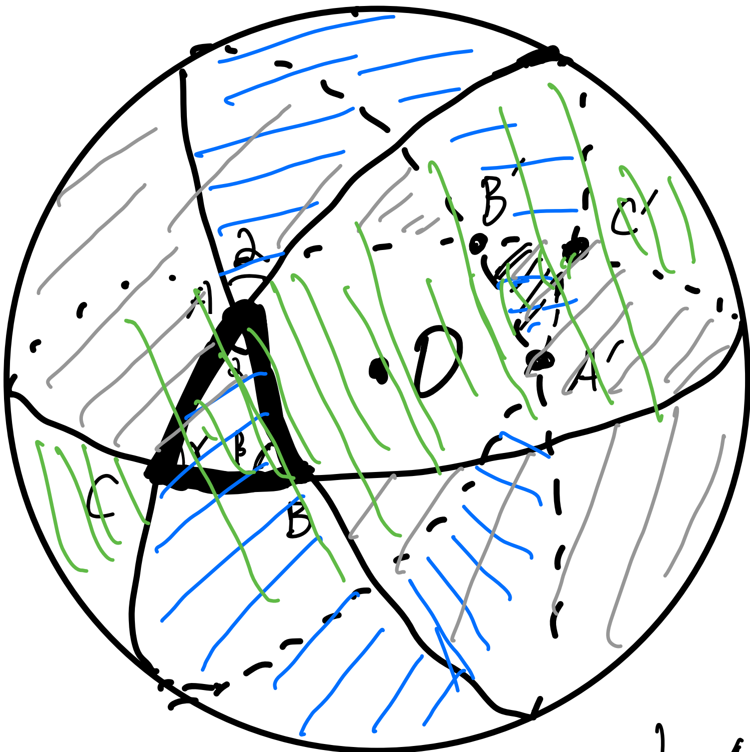






$$\underbrace{4\pi \cdot \frac{\alpha}{2\pi} = 2\alpha}_{\text{Area(luna)}}$$

$L(N, \alpha)$.



$$|\mathcal{L}(A,\alpha)| = 2\alpha$$
$$|\mathcal{L}(B,\beta)| = 2\beta$$
$$|\mathcal{L}(C,\gamma)| = 2\gamma$$

$$\boxed{4|\triangle ABC| + 4\pi = 4\cdot(\alpha+\beta+\gamma).}$$

$$\Downarrow$$

$$|\triangle ABC| = \underbrace{(\alpha+\beta+\gamma-\pi)}_{>0}$$