Quadratic Equations:

$$\boxed{Ax^2+2Bxy+Cy^2}+2Dx+2Ey+F=0.$$

$$\| $$

$$(x\ y)\begin{pmatrix} A & B \\ B & C \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

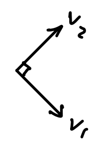
$M=\begin{pmatrix} A & B \\ B & C \end{pmatrix}$ is diagonalizable by an orthogonal matrix $\boxed{S=(v_1\ v_2)}$. $S^{-1}=S^T$

$$S^T S = I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Updownarrow$$

$$|v_1|^2=|v_2|^2=1,\ v_1\cdot v_2=0$$

$$v_1\perp v_2=0$$



$$S^T M S=\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \Leftrightarrow \underset{\underset{\underset{(Mv_1\ Mv_2)}{\|}}{M(v_1\ v_2)}}{\underset{\|}{MS}} = \underset{\underset{\underset{(\lambda_1 v_1\ \lambda_2 v_2)}{\|}}{\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}(v_1\ v_2)}}{\underset{\|}{\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} S}}$$

$$\Leftrightarrow \begin{cases} Mv_1=\lambda_1 v_1 \\ Mv_2=\lambda_2 v_2 \\ v_1\perp v_2,\ |v_1|^2=|v_2|^2=1 \end{cases} \qquad \text{Set } \begin{pmatrix} x \\ y \end{pmatrix} = S\begin{pmatrix} u \\ v \end{pmatrix}.$$

$$\Rightarrow (x\ y)M\begin{pmatrix} x \\ y \end{pmatrix} = (u\ v)S^T M S\begin{pmatrix} u \\ v \end{pmatrix} = (u\ v)\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \underline{\lambda_1 u^2+\lambda_2 v^2}$$

$v_1, v_2 \rightsquigarrow$ axes of conic curve.

Ex: $4x^2-4y^2+6xy\ \underline{-6x-2y+1}=0$.

$$\boxed{M=\begin{pmatrix} 4 & 3 \\ 3 & -4 \end{pmatrix}}. \qquad |\lambda I-M| = \begin{vmatrix} \lambda-4 & -3 \\ -3 & \lambda+4 \end{vmatrix} = \lambda^2-16-9=\lambda^2-25=0$$

$$\lambda=5,-5$$

$$\lambda=5:\ \lambda I-M=\begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix}\to\begin{pmatrix} 1 & -3 \\ 0 & 0 \end{pmatrix}\Rightarrow w_1=\begin{pmatrix} 3 \\ 1 \end{pmatrix}.\Rightarrow v_1=\frac{w_1}{|w_1|}=\underline{\frac{1}{\sqrt{10}}\begin{pmatrix} 3 \\ 1 \end{pmatrix}}.$$

$$\lambda=-5:\ \lambda I-M=\begin{pmatrix} -9 & -3 \\ -3 & -1 \end{pmatrix}\to\begin{pmatrix} 3 & 1 \\ 0 & 0 \end{pmatrix}\Rightarrow w_2=\begin{pmatrix} -1 \\ 3 \end{pmatrix}\Rightarrow v_2=\frac{w_2}{|w_2|}=\underline{\frac{1}{\sqrt{10}}\begin{pmatrix} -1 \\ 3 \end{pmatrix}}.$$

$$S=(v_1\ v_2)=\frac{1}{\sqrt{10}}\begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = S \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\underline{-6x-2y} = (-6\ \ -2)\begin{pmatrix} x \\ y \end{pmatrix} = (-6\ \ -2)\cdot S\begin{pmatrix} u \\ v \end{pmatrix} = \underline{(-6\ \ -2)}\frac{1}{\sqrt{10}}\underline{\begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}}\begin{pmatrix} u \\ v \end{pmatrix}.$$

$$= \frac{1}{\sqrt{10}}(-18-2\ \ \ 6-6)\begin{pmatrix} u \\ v \end{pmatrix} = \frac{1}{\sqrt{10}}(-20\ \ 0)\begin{pmatrix} u \\ v \end{pmatrix} = -\frac{20}{\sqrt{10}}u = \underline{-2\sqrt{10}\cdot u}.$$

$$\rightsquigarrow \boxed{5u^2 - 5v^2 - 2\sqrt{10}\,u + 1 = 0}$$

$$\underline{5\left(u^2 - \frac{2\sqrt{10}}{5}u\right)} - 5v^2 = -1.$$

$$\parallel$$

$$5\left(u^2 - \frac{2\sqrt{10}}{5}u + \frac{10}{25}\right) - 5v^2 = -1 + 2$$

$$\parallel$$

$$5\left(u - \frac{\sqrt{10}}{5}\right)^2 - 5v^2 = 1.$$

$$\left\{\frac{u-u_0}{a} \pm \frac{v-v_0}{b} = 0\right\}$$

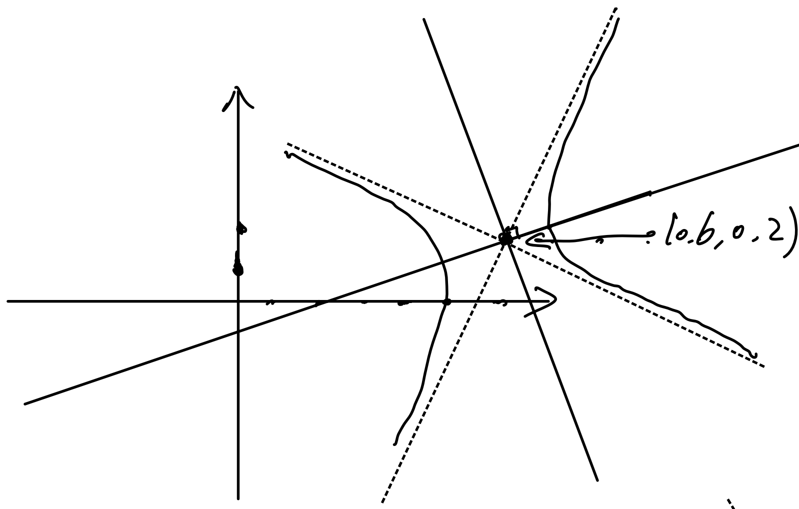
$$\boxed{\frac{(u-u_0)^2}{a^2} - \frac{(v-v_0)^2}{b^2} = 1}$$

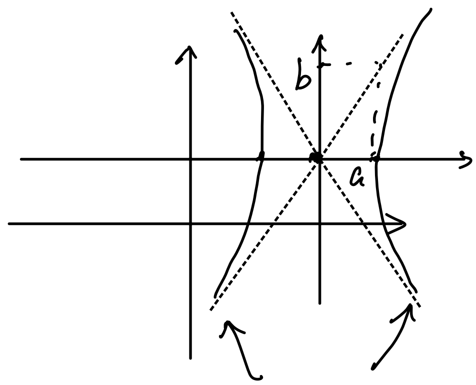
$$\Rightarrow \frac{\left(u - \frac{\sqrt{10}}{5}\right)^2}{\left(\frac{1}{\sqrt{5}}\right)^2} - \frac{v^2}{\left(\frac{1}{\sqrt{5}}\right)^2} = 1.$$

Center : $(u_0, v_0) = \left(\frac{\sqrt{10}}{5}, 0\right).$

$$\rightsquigarrow \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = S\cdot\begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = \frac{1}{\sqrt{10}}\begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}\cdot\begin{pmatrix} \frac{\sqrt{10}}{5} \\ 0 \end{pmatrix} = \frac{1}{\sqrt{10}}\cdot\begin{pmatrix} \frac{3\sqrt{10}}{5} \\ \frac{\sqrt{10}}{5} \end{pmatrix} = \begin{pmatrix} \frac{3}{5} \\ \frac{1}{5} \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.2 \end{pmatrix}.$$

Axis : $v_1 = \frac{1}{\sqrt{10}}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $v_2 = \frac{1}{\sqrt{10}}\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.





$$\frac{(u-u_0)^2}{a^2} - \frac{(v-v_0)^2}{b^2} = 1$$

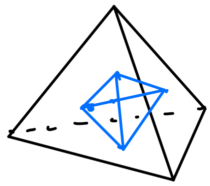
$$\frac{(u-u_0)^2}{a^2} - \frac{(v-v_0)^2}{b^2} = 0$$

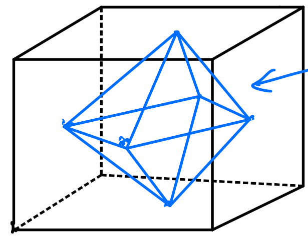
$$\frac{u-u_0}{a} \pm \frac{v-v_0}{b} = 0$$

$$\parallel$$

$$\underline{\left(\frac{u-u_0}{a} + \frac{v-v_0}{b}\right)}\underline{\left(\frac{u-u_0}{a} - \frac{v-v_0}{b}\right)} = 0$$

Platonic Solids = regular polyhedron.





Octahedron.

\# of

Vertices $V = 4$

Edges $E = 6$

Faces $F = 4$

$V = 8$ $\quad$ $V = 6$

$E = 12$ $\quad$ $E = 12$

$F = 6$ $\quad$ $F = 8$

Dodecahedron $\longleftrightarrow$ Icosahedron.

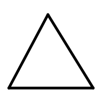
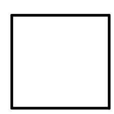
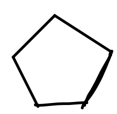
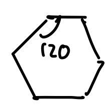
$$\boxed{V - E + F = 2}$$

$\parallel$

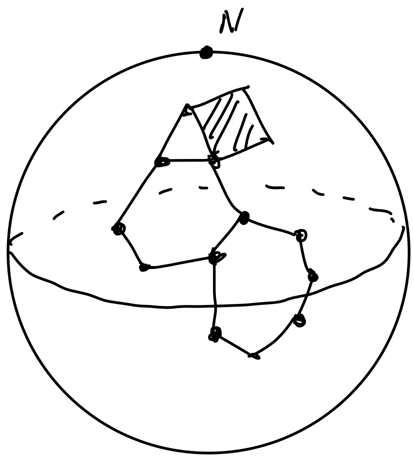
$\chi(S^2)$.

Dodecahedron	Icosahedron
$V = 20$	$V = 12$
$E = 30$	$E = 30$
$F = 12$ 5-gon	$F = 20$ 3-gon

Fact: These are all regular polyhedons.

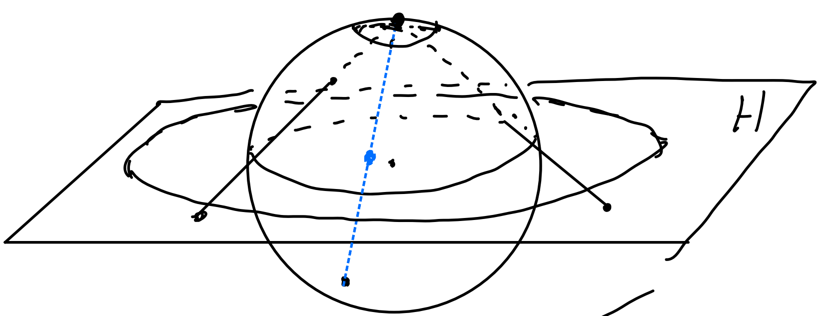
Possible faces:    

# faces at a vertex:	3, 4, 5	3	3
	↑ ↑ ↑	↑	↑
	Te Oc Ic	Cu	Do

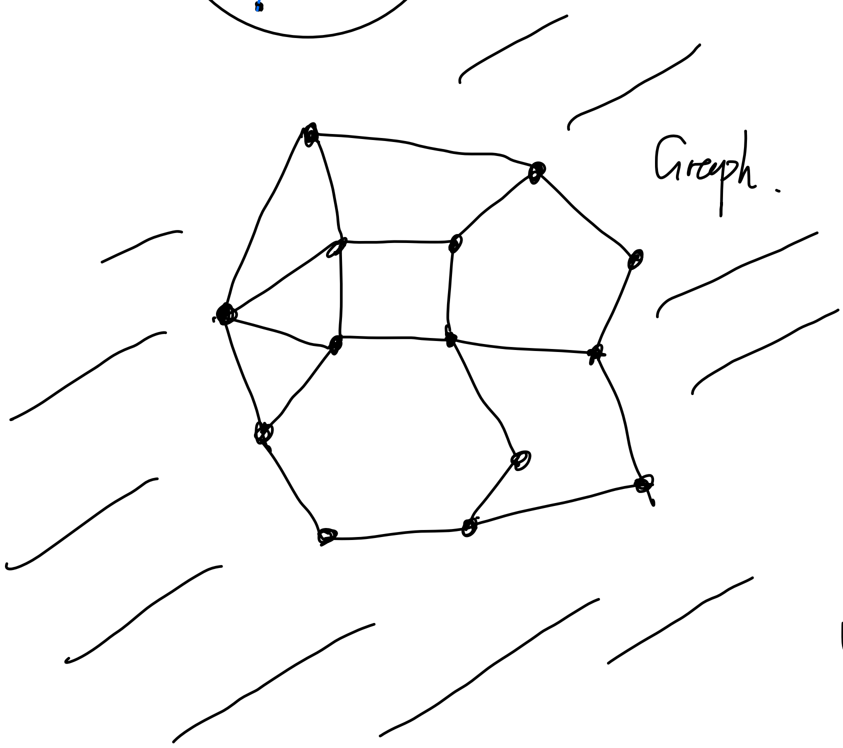


$$\boxed{V - E + F = 2 = \chi(S^2)}$$

Euler characteristic of S^2.



$S^2 \setminus \{N\} \to$ Plane



Graph.

$$\begin{array}{c} V - E + (F-1) \\ \shortparallel \\ 1 \end{array}$$

For a Graph on a plane

$$\begin{array}{ccccc} v - & e & + & f & = 1. \\ \shortparallel & \shortparallel & & \shortparallel & \\ V & E & & (F-1) & \end{array}$$