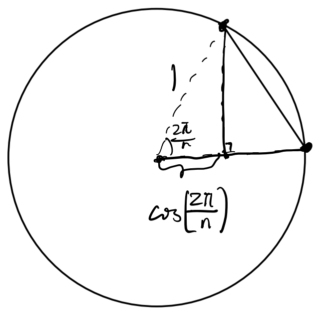
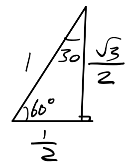
construct regular n-gon $\Longleftrightarrow$ construct $\cos\left(\frac{2\pi}{n}\right)$.



$$\cos(30^\circ) = \frac{\sqrt{3}}{2}, \quad 30^\circ = \frac{\pi}{6}$$



$$\cos(60^\circ) = \frac{1}{2}, \quad 60^\circ = \frac{\pi}{3} = \frac{2\pi}{6}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \quad \frac{\pi}{4} = 45^\circ$$

$$\boxed{2\cdot\cos\theta = \omega + \omega^{-1}} \qquad \theta = \frac{2\pi}{n}. \quad \omega = e^{\frac{2\pi i}{n}}. \quad \omega^n = e^{\frac{2\pi i}{n}\cdot n} = e^{2\pi i} = 1.$$

$$\Updownarrow$$

$$\begin{cases} \omega = e^{i\theta} = \cos\theta + i\sin\theta & \text{Euler's formula} \\ \omega^{-1} = e^{-i\theta} = \cos\theta - i\sin\theta \end{cases}$$

$$0 = \omega^n - 1 = (\omega - 1)\cdot(\omega^{n-1} + \omega^{n-2} + \cdots + 1)$$

$$\Downarrow$$

$$\omega^{n-1} + \omega^{n-2} + \cdots + \omega + 1 = 0.$$

$n=5$: $\quad \omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0.$

$\omega = e^{\frac{2\pi i}{5}} \qquad \omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} = 0. \qquad x = \omega + \omega^{-1}.$

$$\Updownarrow \qquad\qquad x^2 = \omega^2 + 2 + \omega^{-2}$$

$$x^2 + x - 1 = x^2 - 2 + x + 1 = 0. \qquad x = 2\cdot\cos\frac{2\pi}{5}$$

$$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2} \to x = \frac{\sqrt{5}-1}{2} = 2\cdot\cos\frac{2\pi}{5} \Rightarrow \boxed{\cos\frac{2\pi}{5} = \frac{\sqrt{5}-1}{4}}$$

$$\frac{360}{5} = 72^\circ$$

$$\cos 15^\circ = \sqrt{\frac{1+\cos 30^\circ}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

$n=7$: $\quad \omega = e^{\frac{2\pi i}{7}}. \quad \omega^6 + \omega^5 + \omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0.$

$$\omega^3 + \omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} + \omega^{-3} = 0.$$

$$x = \omega + \omega^{-1} = 2\cdot\cos\frac{2\pi}{7} \rightsquigarrow \begin{array}{l} x^3 = \omega^3 + 3\omega + 3\omega^{-1} + \omega^{-3} = \omega^3 + \omega^{-3} + 3x \\ x^2 = \omega^2 + 2 + \omega^{-2} \end{array}$$

$$x^3-3x+x^2-2+x+1=0$$

$$\parallel$$

$$\rightarrow \boxed{x^3+x^2-2x-1=0}$$

$$y^3=10 \Rightarrow y=\sqrt[3]{10}.$$

formula for x involves cubic root $\rightsquigarrow$ x is not constructible!

									2×5		2×6			3×5	2^4	$2^{2^2}+1$
$\cos\left(\frac{2\pi}{n}\right)$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	✓	✓	✓	✓	✓	X	✓	X	✓	X	✓	X	X	✓	✓	✓

$$\cos\theta=\sqrt{\frac{1+\cos(2\theta)}{2}}$$

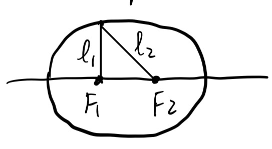
$$\cos\frac{2\pi}{8} \uparrow \cos\frac{2\pi}{4}$$

$$n=2^r\cdot P_1\cdot P_2\cdots P_m$$

$$\cos\left(\frac{2\pi}{17}\right) = \sqrt{\quad\sqrt{\sqrt{\ }}}$$

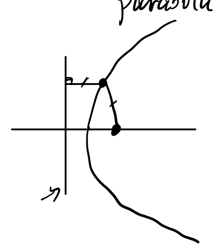
- Conic curves.

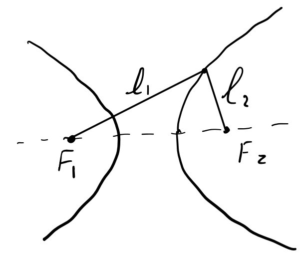
ellipse



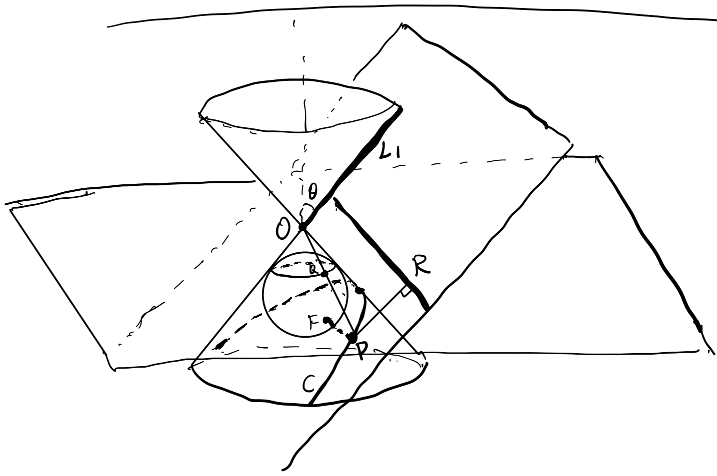
$l_1+l_2=\text{const}.$

parabola

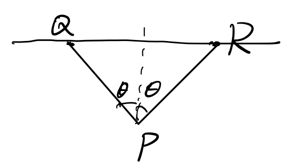


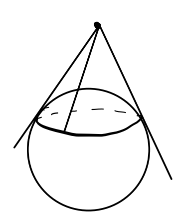


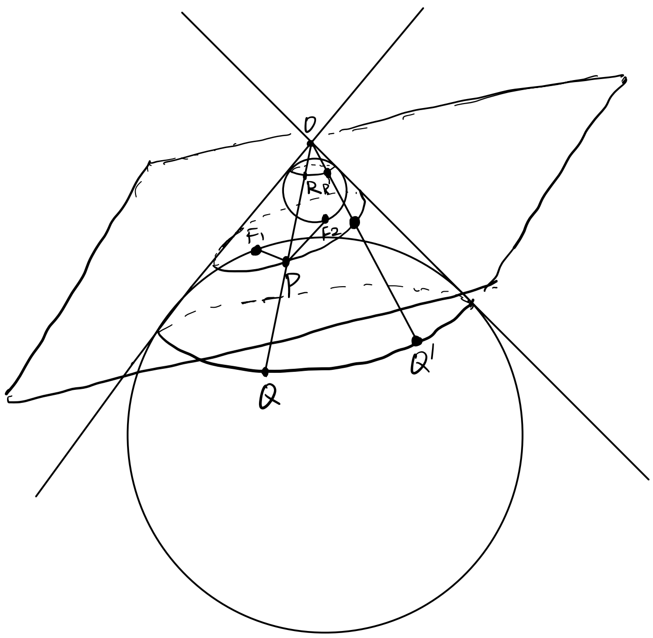
$|l_1-l_2|=\text{const}.$

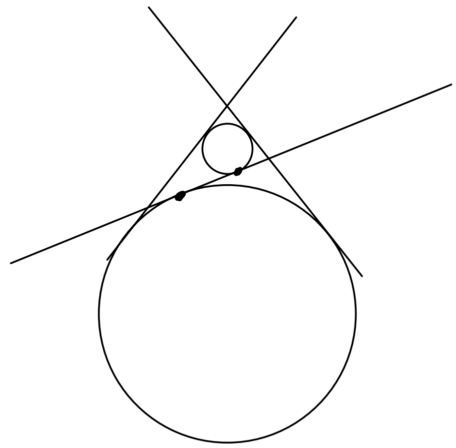


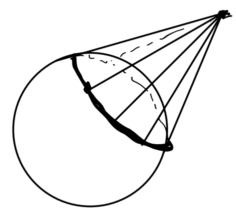
$$|PR|.$$
$$\parallel$$
$$|FP|=|PQ|$$



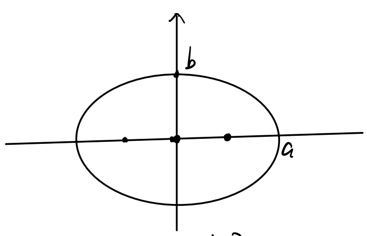




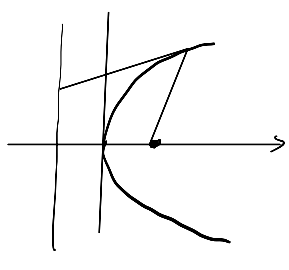




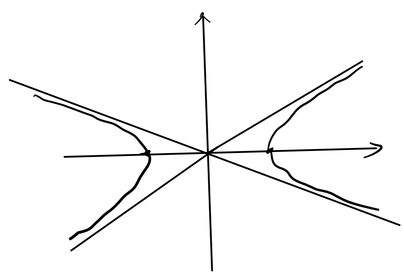
$$|PF_1| = |PQ| \doteq |RQ| = |R'Q'|$$
$$+ \qquad +$$
$$|PF_2| = |PR|$$



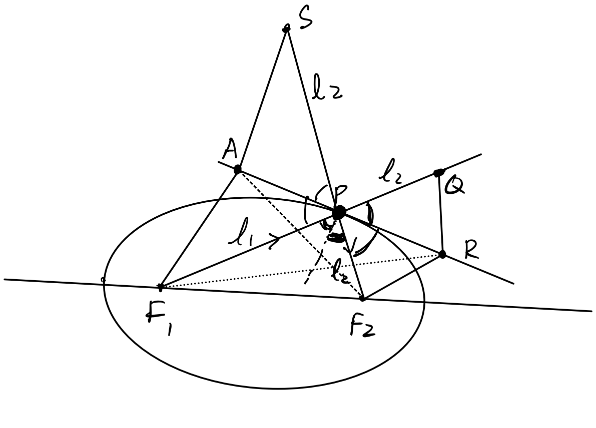
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$x = c \cdot y^2$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$



SAS

$\triangle F_2PR \cong \triangle QPR$

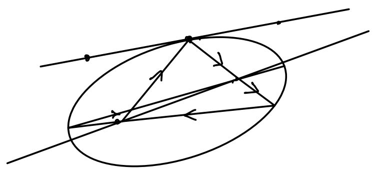
$\Rightarrow |F_2R| = |RQ|$

$l_1 + l_2 = 2a$

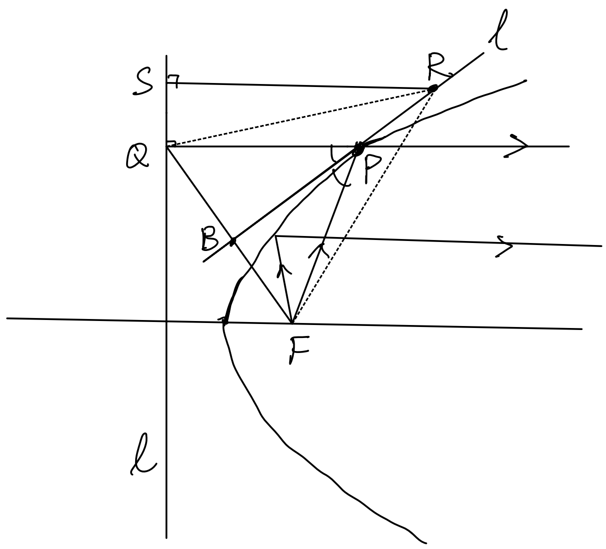
$|F_1R| + |F_2R| = |F_1R| + |RQ| > |F_1Q|$

$\triangle SPA \cong \triangle F_1PA \Rightarrow \underline{|AS| = |F_1A|}$

$|AF_1| + |AF_2| = |AS| + |AF_2| > |SF_2| = 2a.$



l: tangent to parabola at $P \Longleftrightarrow$ bisection of $\angle FPQ$



$\triangle QPB \cong \triangle FPB \Rightarrow \underline{|BQ| = |BF|}$

$\triangle FBR \cong \triangle QBR \Rightarrow \underline{|QR| = |FR|}$

$|QR| > |SR|$

$|RF| = |QR| > |SR|$