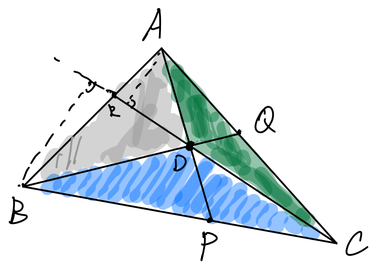
Ceva Thm



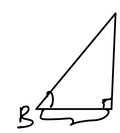
AP, BQ and CR concurrent $\Longleftrightarrow$ $\dfrac{|AR|}{|RB|} \cdot \dfrac{|BP|}{|PC|} \cdot \dfrac{|CQ|}{|QA|} = 1$.

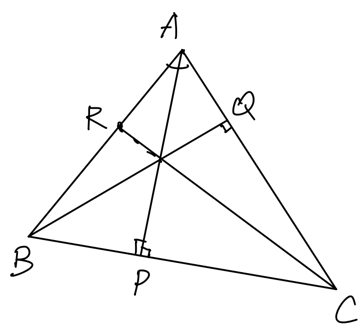
"$\Rightarrow$"

Pf: $$\frac{|AR|}{|RB|} = \frac{|\triangle ARD|}{|\triangle RBD|} = \frac{|\triangle ARC|}{|\triangle RBC|} = \frac{|\triangle ADC| = |\triangle ARD| - |\triangle ARC|}{|\triangle BDC| = |\triangle RBC| - |\triangle RBD|}$$

$$\frac{|BP|}{|PC|} = \frac{|\triangle ABD|}{|\triangle ADC|}, \quad \frac{|CQ|}{|QA|} = \frac{|\triangle BDC|}{|\triangle ABD|}$$

$$\left(\frac{|\triangle_1|}{|\triangle_2|} \cdot \frac{|\triangle_2|}{|\triangle_3|} \cdot \frac{|\triangle_3|}{|\triangle_1|} = 1 \right).$$



- 

$$\frac{|BP|}{|PC|} = \frac{|AB| \cdot \cos B}{|AC| \cdot \cos C}$$

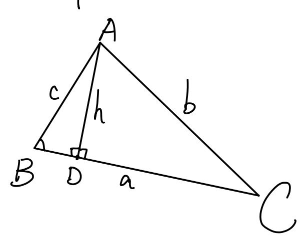
$$\frac{|CQ|}{|QA|} = \frac{|BC| \cdot \cos C}{|AB| \cdot \cos A} \Rightarrow \text{product} = 1.$$

$\Downarrow$

concurrent.

$$\frac{|AR|}{|RB|} = \frac{|AC| \cdot \cos A}{|BC| \cdot \cos B}$$

- Law of Cosine



$h = c \cdot \sin B$

$|DC| = |a - c \cdot \cos B| = |BC| - |BD|$.

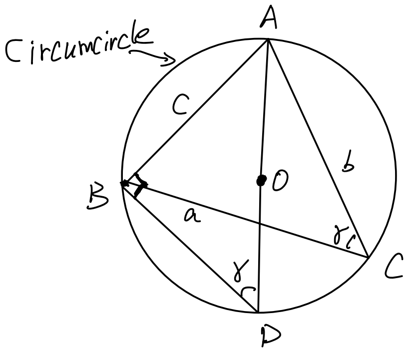
$h^2 + |DC|^2 = b^2$

$$c^2\cdot\sin^2 B + (a - c\cdot\cos B)^2 = b^2$$

$$\|$$

$$c^2\cdot\sin^2 B + a^2 - 2ac\cdot\cos B + c^2\cos^2 B = \boxed{a^2 + c^2 - 2ac\cdot\cos B = b^2}$$

· Law of sine.

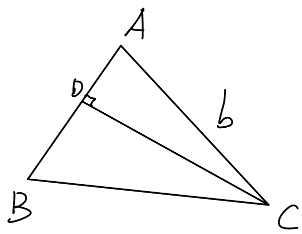


$$c = |AB| = |AD|\cdot\sin C \Rightarrow \frac{c}{\sin C} = 2\cdot R$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}, \qquad 2\cdot R = \frac{a}{\sin A}$$

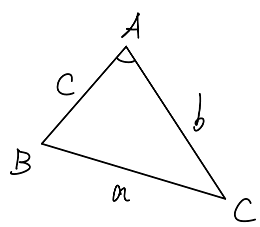
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

· Area formula



$$|\triangle ABC| = |AB|\cdot|CD|\frac{1}{2}$$
$$= |AB|\,|AC|\cdot\sin A\cdot\frac{1}{2}$$
$$= \frac{1}{2}|AB|\,|AC|\cdot\sin A.$$

· Heron's formula



· $|\triangle| = \frac{1}{2}\cdot b\cdot c\cdot \sin A$

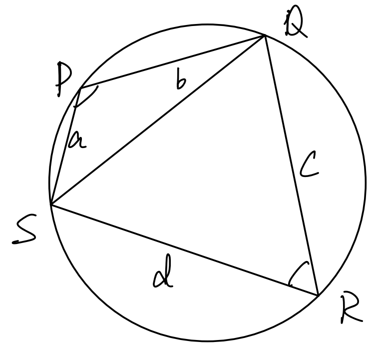
· $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} \Rightarrow \sin A = \sqrt{1 - \cos^2 A}$

$$\Rightarrow |\triangle| = \frac{1}{2}bc\cdot\sqrt{1 - \left(\frac{b^2 + c^2 - a^2}{2bc}\right)^2} = \sqrt{\frac{a+b+c}{2}\cdot\frac{-a+b+c}{2}\cdot\frac{a-b+c}{2}\cdot\frac{a+b-c}{2}}$$

$$|\triangle| = \sqrt{s\cdot(s-a)\cdot(s-b)\cdot(s-c)}. \qquad s = \frac{1}{2}(a+b+c).$$

semiperimeter

$$\frac{1}{2}\cancel{bc}\cdot\frac{1}{2\cancel{bc}}\cdot\sqrt{(2bc)^2-(b^2+c^2-a^2)^2} = \frac{1}{4}\sqrt{\underbrace{(2bc+b^2+c^2-a^2)}_{\substack{\| \\ ((b+c)^2-a^2) \\ \| \\ (b+c+a)\cdot(b+c-a)}}\underbrace{(2bc-b^2-c^2+a^2)}_{\substack{\| \\ a^2-(b-c)^2 \\ \| \\ (a+b-c)(a-b+c)}}}$$



$$|\square PQRS| = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$s = \frac{1}{2}(a+b+c+d).$$

(Brahmagupta's formula).

$\angle P + \angle R = 180°$

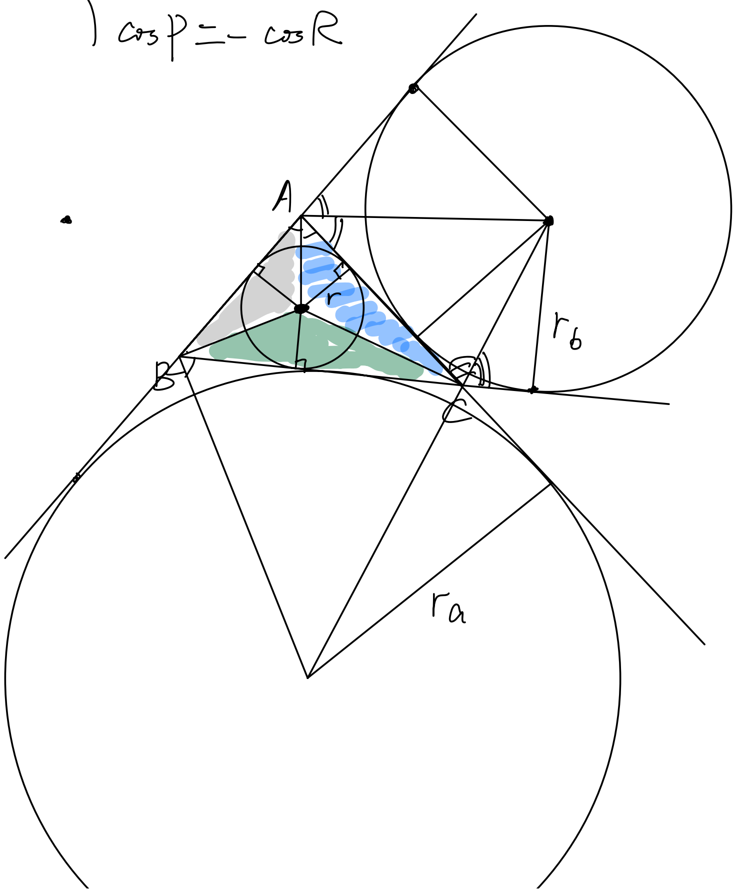
$$\begin{cases} \sin P = \sin R \\ \cos P = -\cos R \end{cases}$$

$$\sin(R) = \sin(\overset{\pi}{\overset{\|}{180°}} - P) = \sin P.$$

$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$

$\sin(\pi-\alpha) = \sin\alpha$

$\cos(\pi-\alpha) = -\cos\alpha.$



$$\Delta = \frac{1}{2}\cdot a\cdot r + \frac{1}{2}b\cdot r + \frac{1}{2}c\cdot r$$

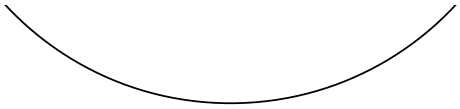
$$= \frac{1}{2}(a+b+c)\cdot r = s\cdot r$$

$$\Delta = (s-a)\cdot r_a$$

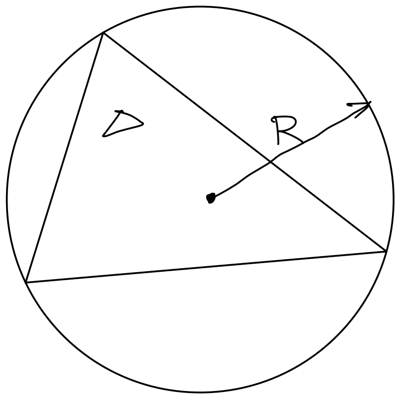
$$= (s-b)\cdot r_b$$

$$= (s-c)\cdot r_c$$

$$\frac{1}{r} = \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c}$$



$$4R = r_a + r_b + r_c - r.$$



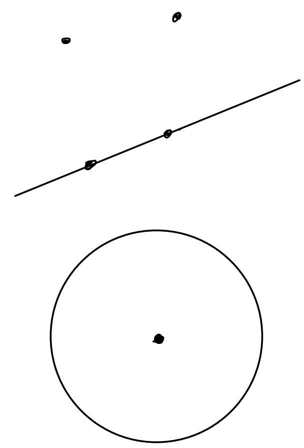
$$R = \frac{a \cdot b \cdot c}{|\Delta|}$$

Construction using compass and straightedge.

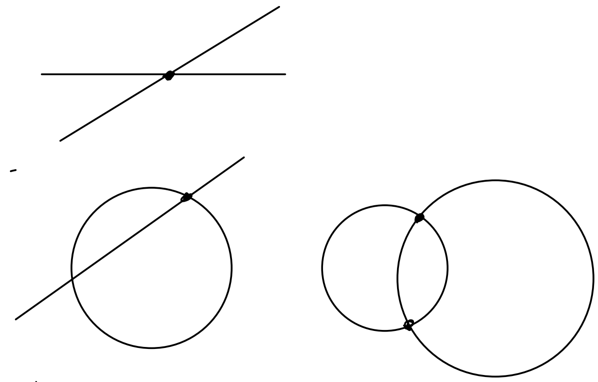
0.

1.

2.

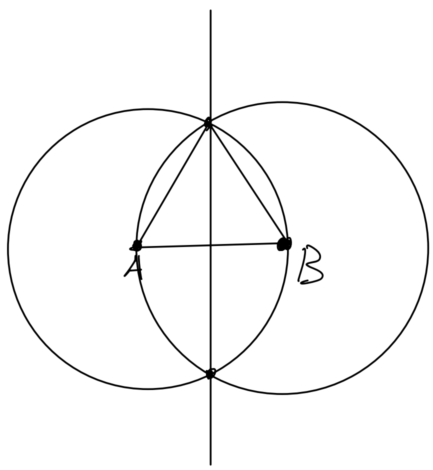


3.

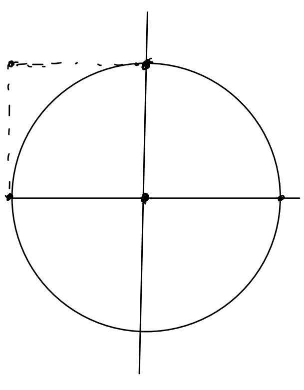


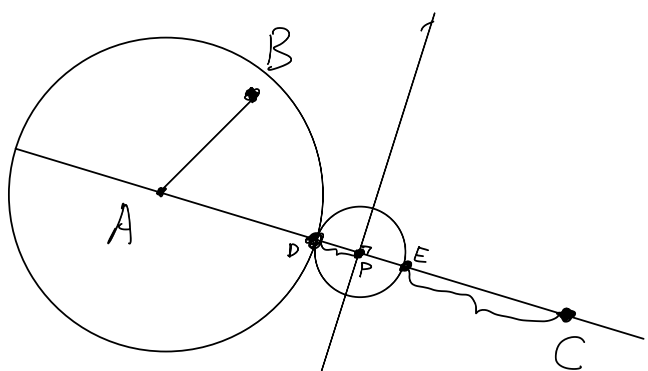
. Equilateral triangle

equidistant line

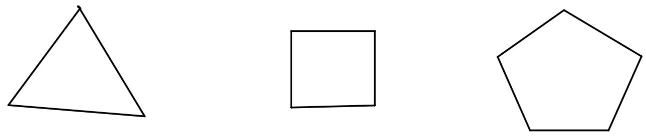


.





$$|EC| = |PC| - |PE|$$
$$= |PA| - |PD|$$
$$= |AD| = |AB|.$$



regular n-polygons is constructible if and only if

$n = 2^r \cdot p_1 \cdot p_2 \cdots p_k$. prime factorization

where $p_i = 2^{2^r} + 1$ Fermat primes.

$r=0$: $2^{2^0}+1 = 2^1+1 = 3$

$r=1$: $2^{2^1}+1 = 5$.

$r=2$: $2^{2^2}+1 = 2^4+1 = 17$. (Gauss)

$r=3$: $2^{2^3}+1 = 2^8+1 = 257$

$r=4$: $2^{2^4}+1 = 2^{16}+1 = 65537$.

$r=5$. $2^{2^5}+1 = 641 \times 6700417$ not prime!

No other known Fermat prime at this time.