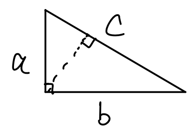
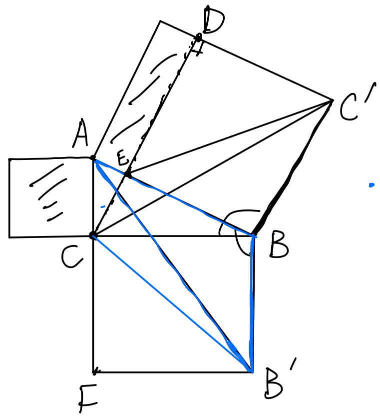
Pythagorean Thm:



$a^2+b^2=c^2$



- $\triangle CBC' \cong \triangle B'BA$

$|C'B| = |AB|$

$\angle C'BC = \angle ABB'$

$|CB| = |B'B|$

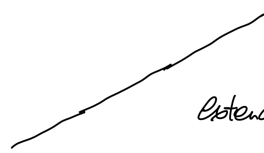
$\Big\} \Uparrow$ SAS

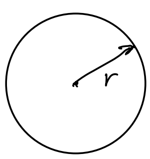
- $|\triangle B'BA| = |\triangle B'BC|$, $|\triangle CBC'| = |\triangle EBC'|$

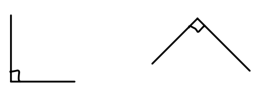
$\Rightarrow$ $|\square BC'DE| = |\square B'BCF|$

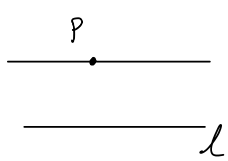
Axioms Euclidean Geometry.

1. 

2.  extend indefinitely

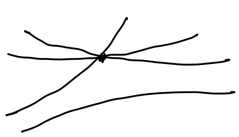
3. 

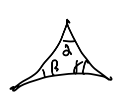
4.  $90^\circ = \frac{\pi}{2}$

5. 

existence of parallel line and it is unique.

(Playfair's Version)

Later in hyperbolic geometry 

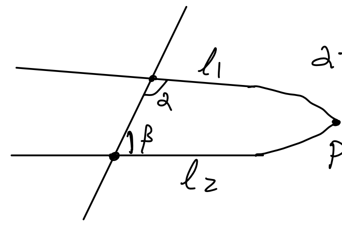


$\alpha+\beta+\gamma < \pi = 180^\circ$

Euclid: Elements of Geometry.

Axiom 5 $\Longleftrightarrow$ Euclidean version :
(Playfair)

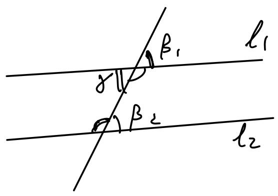
$\Downarrow$



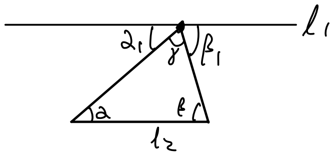
$\alpha+\beta < \underset{\pi}{180^\circ} = 2\times 90^\circ$

$\Downarrow$

$l_1 \cap l_2 = P$



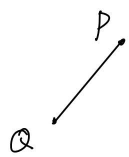
$l_1 // l_2 \Rightarrow \beta_1 = \beta_2\ ,\ \beta_2 = \gamma$



$\alpha+\beta+\gamma = \alpha_1+\beta+\gamma_1 = 180^\circ$.

- 

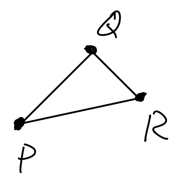
Isometry of plane



$d(P,Q) = |PQ|$ distance

- $d(P,Q) = d(Q,P)$
- $d(P,Q) \geq 0$, and "$=0$" iff $P=Q$.

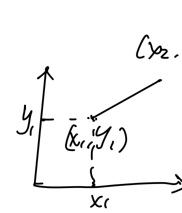
  iff = if and only if
- $d(P,R) \leq d(P,Q) + d(Q,R)$ (triangle inequality).



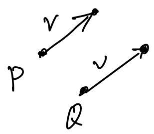
- A map $f: \mathbb{R}^2 \to \mathbb{R}^2$ is an isometry if $d(f(P), f(Q)) = d(P,Q)$.

  $\mathbb{R}^2$ = plane = $\{(x,y) : x,y \in \mathbb{R}^2\}$

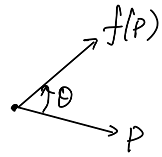
  $d((x_1,y_1),(x_2,y_2)) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$



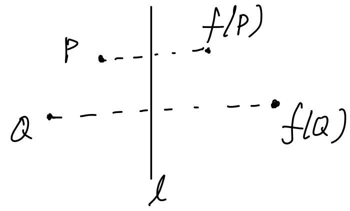
translation



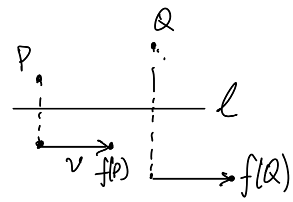
rotation



reflection



glide reflection



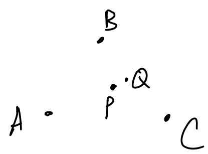
reflection reverses the orientation.

orientation

right handed / left handed

Thm: Any isometry must be one of the above 4 types.

Exercise:



$$\left.\begin{aligned} |PA| &= |QA| \\ |PB| &= |QB| \\ |PC| &= |QC| \end{aligned}\right\} \Rightarrow P = Q.$$