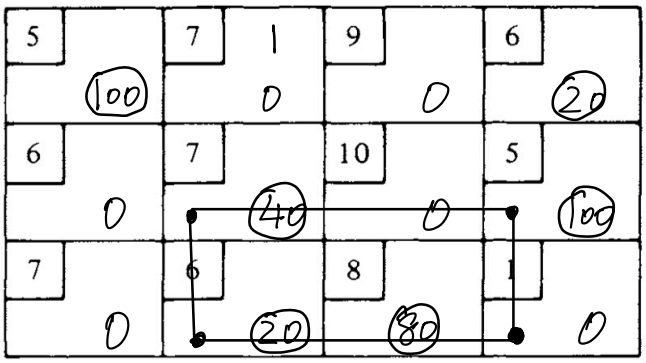
Tableau 5.1



v_1

v_2 Supply

v_3

w_1 w_2 w_3 w_4

$v_1 + w_1 = 5$

$v_1 + w_4 = 6$

$v_2 + w_2 = 7$ ⇒

$v_2 + w_4 = 5$

$v_3 + w_2 = 6$

$v_3 + w_3 = 8$

$v_1 = 0$

$w_1 = 5$

$w_4 = 6$

$v_2 = -1$

$w_2 = 8$

$v_3 = 6 - 8 = -2$

$w_3 = 8 - (-2) = 10$.

x_{12}: $z_{12} - c_{12} = v_1 + w_2 - c_{12} = 0 + 8 - 7 = 1$

x_{13}: $v_1 + w_3 - c_{13} = 0 + 10 - 9 = 1$.

x_{21}: $v_2 + w_1 - c_{21} = -1 + 5 - 6 = -2$.

x_{23}: $v_2 + w_3 - c_{23} = -1 + 10 - 10 = -1$

x_{31}: $v_3 + w_1 - c_{31} = -2 + 5 - 7 = -4$

x_{34}: $v_3 + w_4 - c_{34} = -2 + 6 - 1 = 3$.

⇝ Entering var. = most positive non-basic cell = x_{34}

In[1]:=

```
(*Input: A=Cost matrix, B=Position of basic cells,
IA=Shipping amount corresponding to the postion in B*)
(*Output:matrix of M1=z_{ij}-c_{ij},S=Matrix of Shipping plan*)
TObj[A_, B_, IA_] := Module[{M1, m, n, M, b, i, j, S},
m = Dimensions[A][[1]]; n = Dimensions[A][[2]];
M1 = Table[0, m, n];
S = M1;
M = Table[0, m + n - 1, m + n];
b = Table[0, m + n - 1];
For[i = 1, i ≤ m + n - 1, i++, M[[i, B[[i, 1]]]] = 1;
    M[[i, m + B[[i, 2]]]] = 1;
    b[[i]] = Extract[A, B[[i]]]; (*A[[B[[i]][[1]],B[[i]][[2]]]*)];
V = LinearSolve[M, b];
For[i = 1, i ≤ m, i++,
For[j = 1, j ≤ n, j++,
If[MemberQ[B, {i, j}], M1[[i, j]] = bp;
      S[[i, j]] = IA[[Flatten[Position[B, {i, j}]][[1]]]],
M1[[i, j]] = V[[i]] + V[[m + j]] - A[[i, j]];
]
]
];
{M1 // MatrixForm, S // MatrixForm}]
(*Lp=cells along loops, Bp=Basic cells, Ia=Shipping amount, Mv=Cost matrix*)
(*Output:Bp2=basic position after pivoting, Ia2=new shipping amount, Tc=Shipping cost*)
TPivot[Lp_, Bp_, Ia_, Mv_] := Module[{ll, lpp, elia, pma, ma, Bp2, Ia2, Tc},
ll = Length[Lp];
lpp = Flatten[Table[Position[Bp, Lp[[i]]], {i, 2, Length[Lp]}]]; (*Position in of LP(i) in BP*)
elia = Table[Ia[[ lpp[[2 i - 1]] ]], {i, 1, ll / 2}];
  (*removing the entering var, extract the odd position amount*)
pma = PositionSmallest[elia][[1]];
  (*find the position of smallest amount which is the departing var*)
ma = Min[elia]; (*the smallest amount*)
Bp2 = ReplaceAll[Bp, Lp[[2 * pma]] → Lp[[1]] ;
  (*replacing the even departing position by the entering position*)
lpp = Join[lpp, Position[Bp, Lp[[2 * pma]]][[1]]];
Ia2 = Ia;
For[i = 1, i ≤ ll / 2, i++,
Ia2[[lpp[[2 i - 1]]]] = Ia[[lpp[[2 i - 1]]]] - ma;
Ia2[[lpp[[2 i]]]] = Ia2[[lpp[[2 i]]]] + ma;
];
Tc = Sum[Extract[Mv, Bp2[[i]]] * Ia2[[i]], {i, 1, Length[Ia2]}];
```

```
{Bp2, Ia2, Tc}
]
```

In[3]:=
```
CM = {{5, 7, 9, 6}, {6, 7, 10, 5}, {7, 6, 8, 1}};
T1a = {{{1, 1}, {1, 4}, {2, 2}, {2, 4}, {3, 2}, {3, 3}},
{100, 20, 40, 100, 20, 80}};
T1 = Join[T1a, {Sum[Extract[CM, T1a[[1]][[i]]] * T1a[[2]][[i]], {i, 1, Length[T1a[[2]]}]}]
TObj[CM, T1[[1]], T1[[2]]]
```

Out[5]= {{{1, 1}, {1, 4}, {2, 2}, {2, 4}, {3, 2}, {3, 3}}, {100, 20, 40, 100, 20, 80}, 2160}

Out[6]=
$$\left\{\begin{pmatrix} bp & 1 & 1 & bp \\ -2 & bp & -1 & bp \\ -4 & bp & bp & 3 \end{pmatrix}, \begin{pmatrix} 100 & 0 & 0 & 20 \\ 0 & 40 & 0 & 100 \\ 0 & 20 & 80 & 0 \end{pmatrix}\right\}$$

In[7]:=
```
T2 = TPivot[{{3, 4}, {3, 2}, {2, 2}, {2, 4}}, T1[[1]], T1[[2]], CM]
TObj[CM, T2[[1]], T2[[2]]]
```

Out[7]= {{{1, 1}, {1, 4}, {2, 2}, {2, 4}, {3, 4}, {3, 3}}, {100, 20, 60, 80, 20, 80}, 2100}

Out[8]=
$$\left\{\begin{pmatrix} bp & 1 & 4 & bp \\ -2 & bp & 2 & bp \\ -7 & -3 & bp & bp \end{pmatrix}, \begin{pmatrix} 100 & 0 & 0 & 20 \\ 0 & 60 & 0 & 80 \\ 0 & 0 & 80 & 20 \end{pmatrix}\right\}$$

In[9]:=
```
T3 = TPivot[{{1, 3}, {1, 4}, {3, 4}, {3, 3}}, T2[[1]], T2[[2]], CM]
TObj[CM, T3[[1]], T3[[2]]]
```

Out[9]= {{{1, 1}, {1, 3}, {2, 2}, {2, 4}, {3, 4}, {3, 3}}, {100, 20, 60, 80, 40, 60}, 2020}

Out[10]=
$$\left\{\begin{pmatrix} bp & -3 & bp & -4 \\ 2 & bp & 2 & bp \\ -3 & -3 & bp & bp \end{pmatrix}, \begin{pmatrix} 100 & 0 & 20 & 0 \\ 0 & 60 & 0 & 80 \\ 0 & 0 & 60 & 40 \end{pmatrix}\right\}$$

In[13]:=
```
T4 = TPivot[{{2, 3}, {2, 4}, {3, 4}, {3, 3}}, T3[[1]], T3[[2]], CM]
TObj[CM, T4[[1]], T4[[2]]]
```

Out[13]=
{{{1, 1}, {1, 3}, {2, 2}, {2, 4}, {3, 4}, {2, 3}}, {100, 20, 60, 20, 100, 60}, 1900}

Out[14]=
$$\left\{\begin{pmatrix} bp & -1 & bp & -2 \\ 0 & bp & bp & bp \\ -5 & -3 & -2 & bp \end{pmatrix}, \begin{pmatrix} 100 & 0 & 20 & 0 \\ 0 & 60 & 60 & 20 \\ 0 & 0 & 0 & 100 \end{pmatrix}\right\}$$