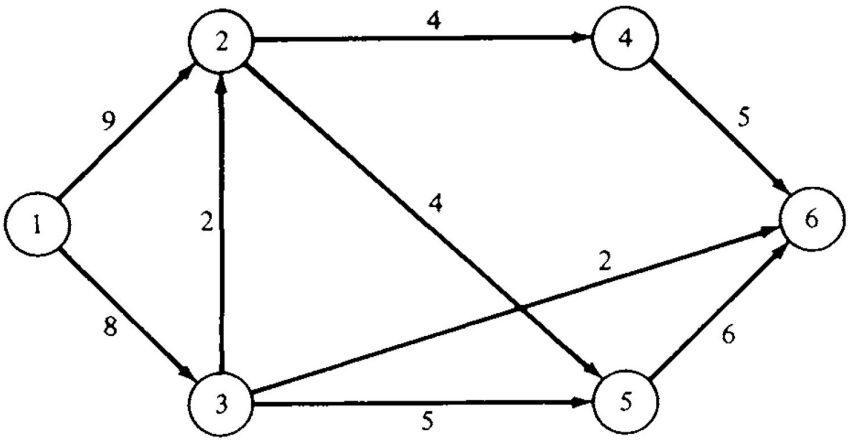

1
2
3
4
5
6
9
8
2
4
4
5
2
6
5

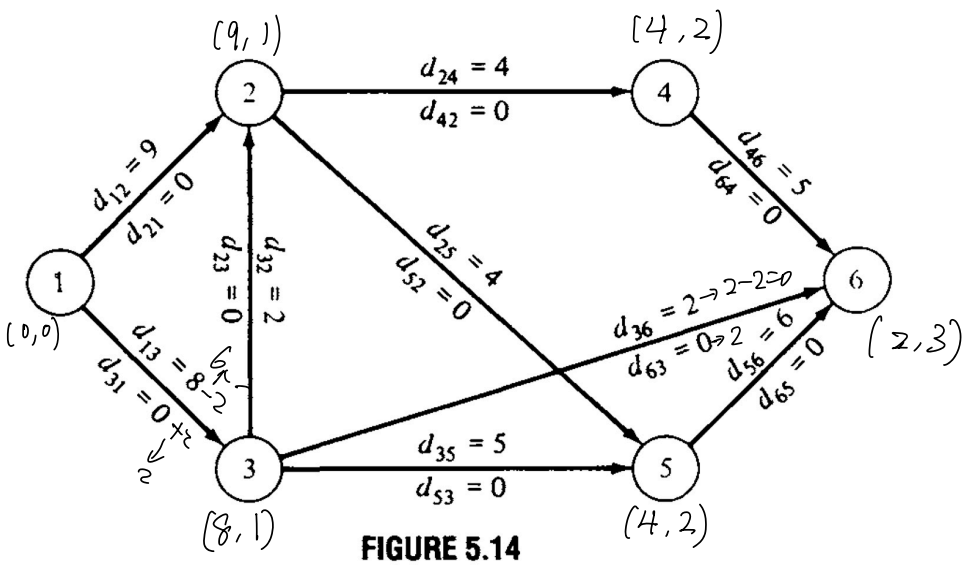
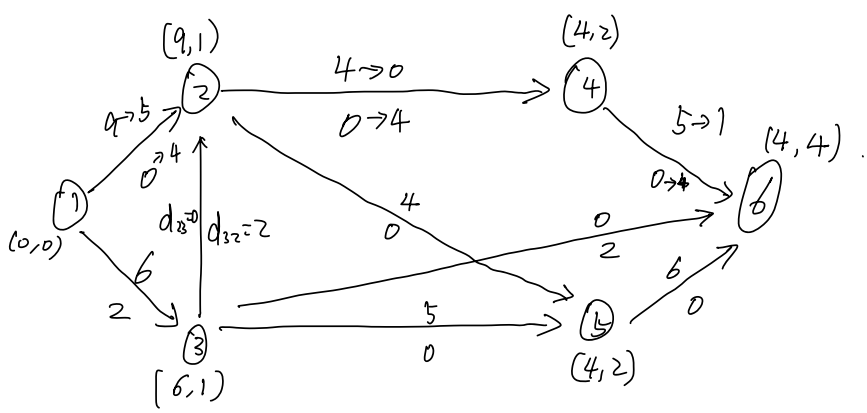
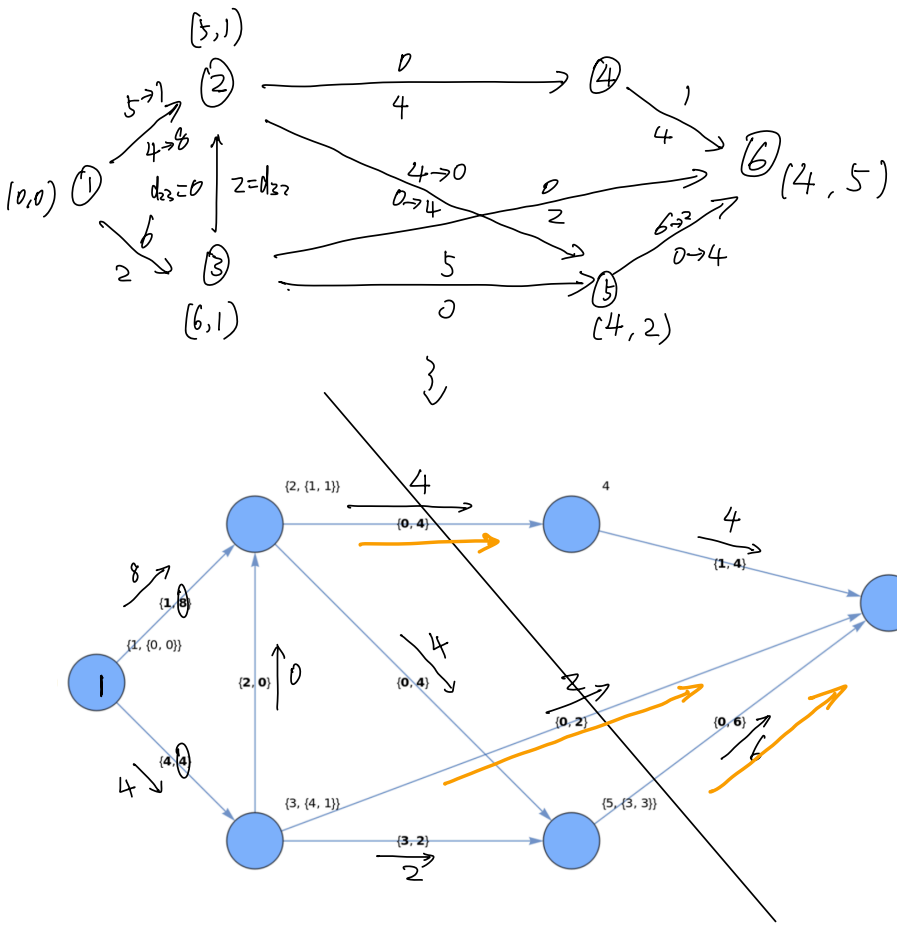

(9,1)
(4,2)
(0,0)
(8,1)
(4,2)
(2,3)
$d_{24} = 4$
$d_{42} = 0$
$d_{12} = 9$
$d_{21} = 0$
$d_{13} = 8$
$d_{31} = 0$
$d_{23} = 0$
$d_{32} = 2$
$d_{25} = 4$
$d_{52} = 0$
$d_{46} = 5$
$d_{64} = 0$
$d_{36} = 2 \to 2-2=0$
$d_{63} = 0 \to 2 = 6$
$d_{56} = 0$
$d_{65} = 0$
$d_{35} = 5$
$d_{53} = 0$

FIGURE 5.14

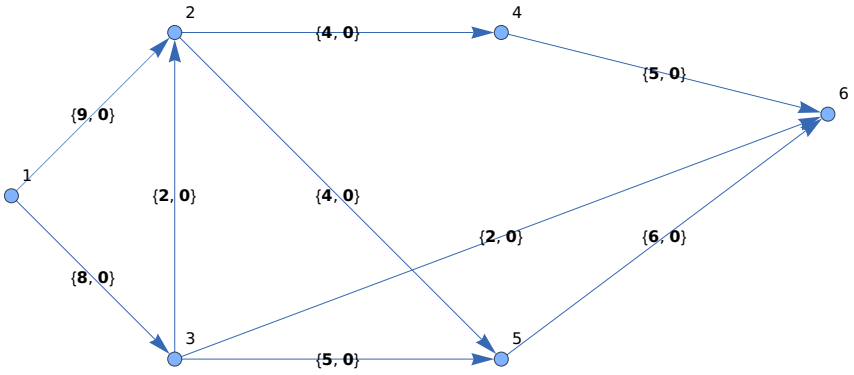

(9,1)
(4,2)
(4,4)
(0,0)
(6,1)
(4,2)
$4 \to 0$
$0 \to 4$
$5 \to 1$
$0 \to 4$
d_{23}
$d_{32} = 2$
4
0
0
2
5
0
6
0



Thm: Maximum flow = minimum of capacities of all cuts.

In[1]:=
```
EPC = {{{1, 2}, 9}, {{1, 3}, 8}, {{2, 4}, 4}, {{2, 5}, 4}, {{3, 2}, 2}, {{3, 5}, 5}, {{3, 6}, 2}, {{4, 6}, 5},
{{5, 6}, 6}};
Ne = Length[EPC];
El = Table[EPC[[i]][[1, 1]] → EPC[[i]][[1, 2]], {i, 1, Ne}];
Cl1 = Table[{EPC[[i]][[2]], 0}, {i, 1, Ne}];
Lel1 = Table[
Labeled[El[[i]], Cl1[[i]]], {i, 1, Ne}];
VCL = {1 → {-2, 0}, 2 → {-1, 1}, 3 → {-1, -1}, 4 → {1, 1}, 5 → {1, -1}, 6 → {3, 1/2}};
G0 = Graph[Lel1, VertexLabels → {"Name"}, EdgeLabelStyle → {Bold}, VertexCoordinates → VCL]
Vl = VertexList[G0];
N11 = {{1, {0, 0}}, {2, {9, 1}}, {3, {8, 1}}};
```

Out[7]=



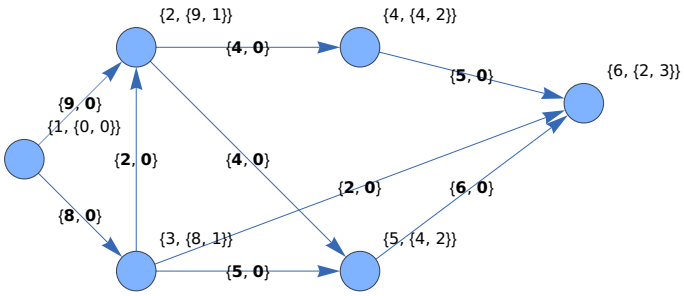
In[21]:=
```
Flow[LVD_, CL_] := Module[{i, j, p, NLVD},
NL = Length[LVD]; (*number of labeled vertices*)
LV = Table[LVD[[i]][[1]], {i, 1, NL}]; (*labeled vertices*)
CL1 = Table[CL[[k]][[1]], {k, 1, Length[CL]}]; (*forward capacity list*)
CL2 = Table[CL[[k]][[2]], {k, 1, Length[CL]}]; (*backward capacity list*)
NLVD = LVD;
For[i = 2, i ≤ NL, i++,
VO = Sort[VertexOutComponent[G0, LV[[i]], {1}]];
VI = Sort[VertexInComponent[G0, LV[[i]], {1}]];
For[j = 1, j ≤ Length[VO], j++,
p = Flatten[Position[El, LV[[i]] → VO[[j]]]][[1]]; (*Position of new edge out of LV[i]*)
If[FreeQ[LV, VO[[j]]] && CL1[[p]] > 0,
NLVD = Join[NLVD, {{VO[[j]], {Min[CL1[[p]], LVD[[i, 2]][[1]], LV[[i]]}}}];
LV = Join[LV, {VO[[j]]}];
];
];
For[j = 1, j ≤ Length[VI], j++,
p = Flatten[Position[El, VI[[j]] → LV[[i]]]][[1]];
If[FreeQ[LV, VI[[j]]] && CL2[[p]] > 0,
NLVD = Join[NLVD, {{VI[[j]], {Min[CL2[[p]], LVD[[i, 2]][[2]], LV[[i]]}}}];
```

```
LV = Join[LV, {VI[[j]]}]
];
];
];
NLVD
]
Aug[FP_, CL_, d_] := Module[{i, p, NCL},
NCL = CL;
For[i = 1, i ≤ Length[FP] - 1, i++,
If[MemberQ[El, FP[[i + 1]] → FP[[i]]],
p = Flatten[Position[El, FP[[i + 1]] → FP[[i]]]][[1]];
NCL[[p]][[1]] = NCL[[p]][[1]] - d;
NCL[[p]][[2]] = NCL[[p]][[2]] + d,
p = Flatten[Position[El, FP[[i]] → FP[[i + 1]]]][[1]];
NCL[[p]][[1]] = NCL[[p]][[1]] + d;
NCL[[p]][[2]] = NCL[[p]][[2]] - d;
]
];
NCL
]
Vld[VLD_] := Module[{i, VLL, UVL, V1, V2},
VLL = Table[VLD[[i]][[1]], {i, Length[VLD]}];
UVL = Complement[Vl, VLL];
V1 = Table[VLD[[i]][[1]] → ToString[{VLD[[i]][[1]], VLD[[i]][[2]]}], {i, 1, Length[VLD]}];
V2 = Table[UVL[[j]] → "Name", {j, 1, Length[UVL]}];
Join[V1, V2]
]
```

In[54]:=
```
N12 = Flow[N11, Cl1] (*new labeled vertex list*)
Vll1 = Vld[N12];
Graph[Lel1, EdgeLabelStyle → {Bold},
 VertexCoordinates → VCL, VertexLabels → Vll1, VertexSize → 0.25]
N11 = N12;
```

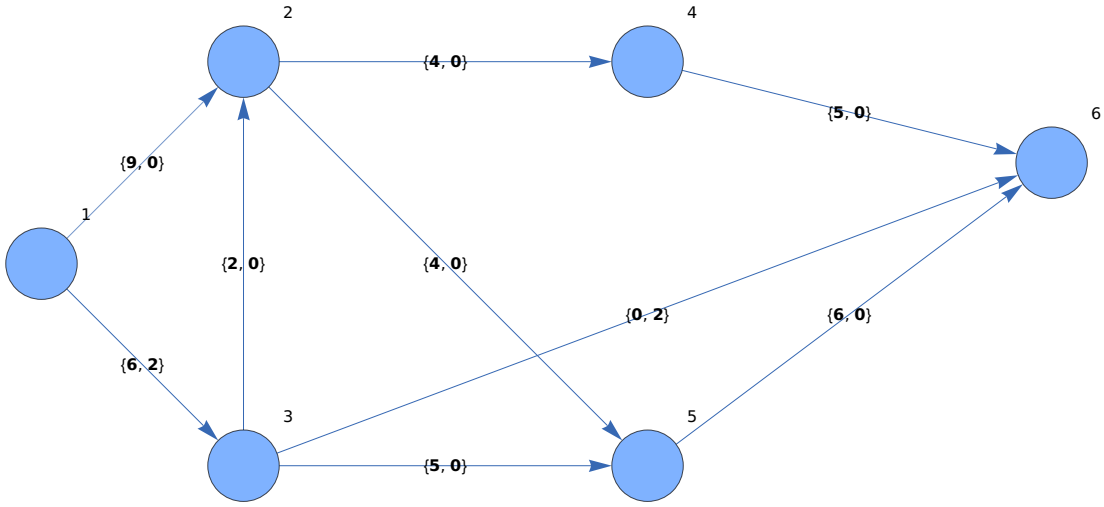
Out[54]=
```
{{1, {0, 0}}, {2, {9, 1}}, {3, {8, 1}}, {4, {4, 2}}, {5, {4, 2}}, {6, {2, 3}}}
```

Out[56]=



In[41]:=
```
Fp1 = {6, 3, 1};
Cl2 = Aug[Fp1, Cl1, 2]; (*Augmented the flow by amount 2*)
Lel2 = Table[
Labeled[El[[i]], Cl2[[i]]], {i, 1, Length[El]}];
Graph[Lel2, VertexLabels → {"Name"}, EdgeLabelStyle → {Bold},
 VertexCoordinates → VCL, VertexLabels → Vll1, VertexSize → 0.25]
```

Out[44]=

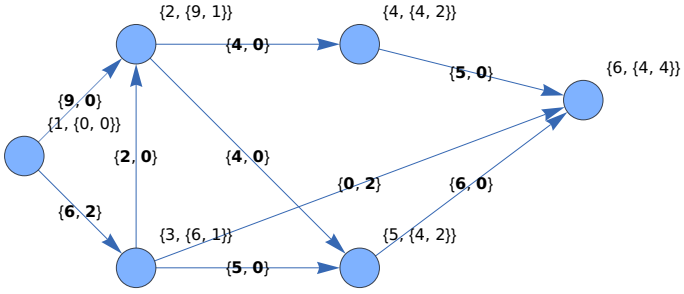


In[59]:=
```
N21 = {{1, {0, 0}}, {2, {9, 1}}, {3, {6, 1}}};
```

In[64]:=
```
(*Run this code twice to label the sink*)
N22 = Flow[N21, Cl2]
Vll2 = Vld[N22];
Graph[Lel2, EdgeLabelStyle → {Bold},
 VertexCoordinates → VCL, VertexLabels → Vll2, VertexSize → 0.25]
N21 = N22;
```

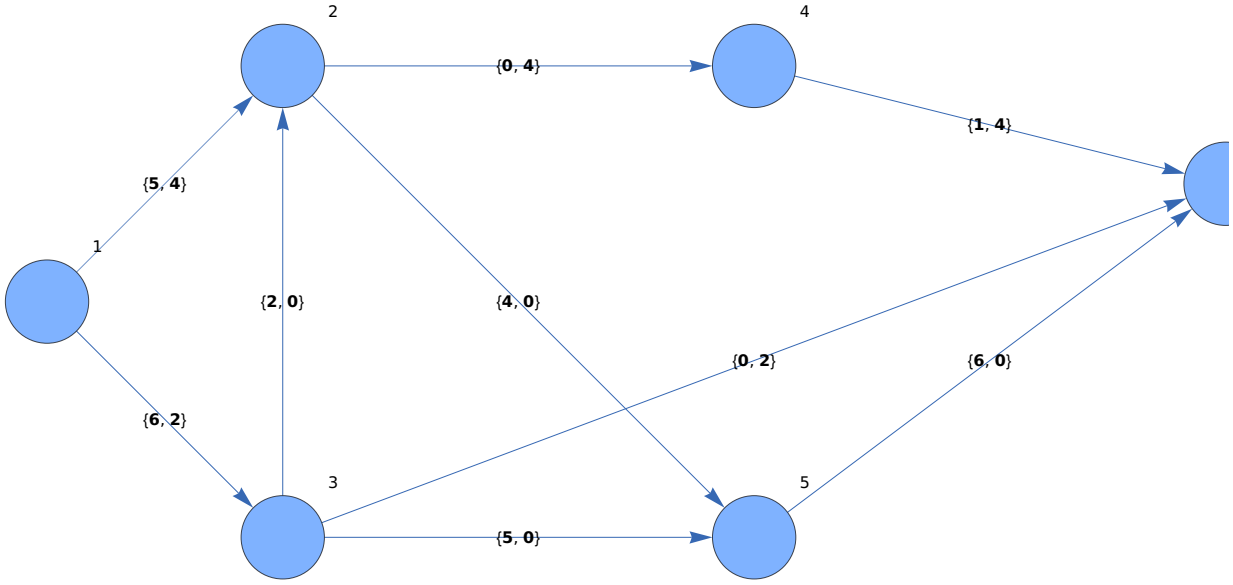
Out[64]=
```
{{1, {0, 0}}, {2, {9, 1}}, {3, {6, 1}}, {4, {4, 2}}, {5, {4, 2}}, {6, {4, 4}}}
```

Out[66]=



In[72]:=
```
Fp2 = {6, 4, 2, 1}; (*path backward to the source*)
Cl3 = Aug[Fp2, Cl2, 4]; (*Augment the flow by amount 4*)
Lel3 = Table[
Labeled[El[[i]], Cl3[[i]]], {i, 1, Length[El]}];
Graph[Lel3, VertexLabels → {"Name"}, EdgeLabelStyle → {Bold},
 VertexCoordinates → VCL, VertexLabels → Vll, VertexSize → 0.25]
```

Out[75]=



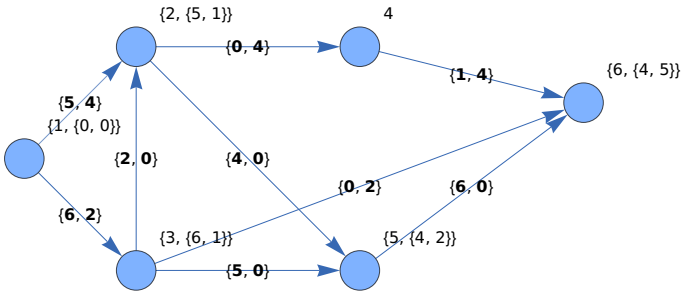
In[76]:=
```
N31 = {{1, {0, 0}}, {2, {5, 1}}, {3, {6, 1}}};
```

In[81]:=
```
N32 = Flow[N31, Cl3]
Vll3 = Vld[N32];
Graph[Lel3, EdgeLabelStyle → {Bold},
 VertexCoordinates → VCL, VertexLabels → Vll3, VertexSize → 0.25]
N31 = N32;
```

Out[81]=

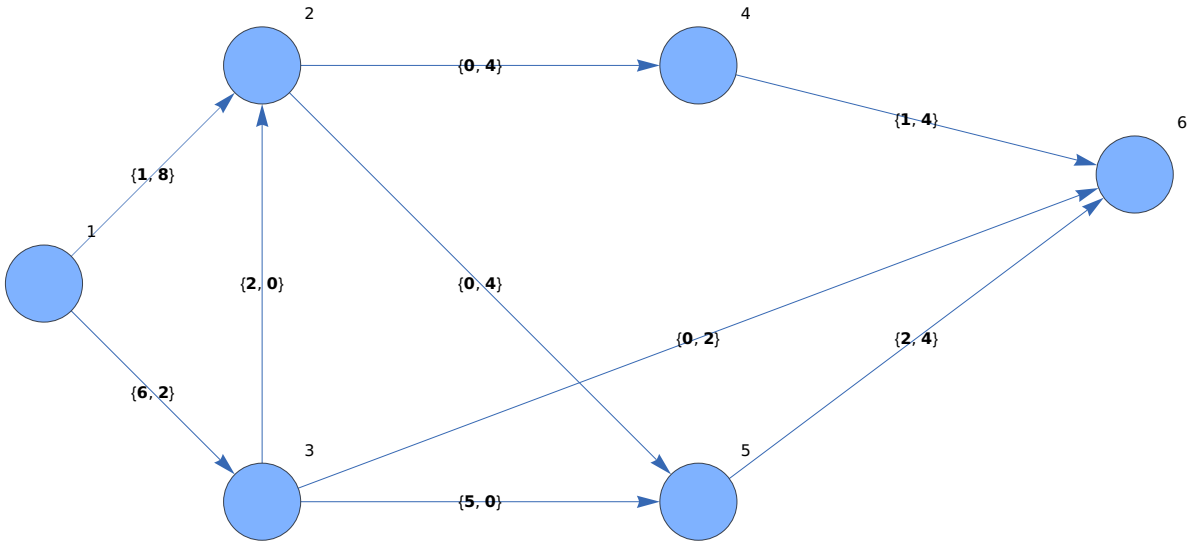
{{1, {0, 0}}, {2, {5, 1}}, {3, {6, 1}}, {5, {4, 2}}, {6, {4, 5}}}

Out[83]=



In[85]:=
```
Fp3 = {6, 5, 2, 1};
Cl4 = Aug[Fp3, Cl3, 4];
Lel4 = Table[
Labeled[El[[i]], Cl4[[i]]], {i, 1, Length[El]}];
G3b = Graph[Lel4, VertexLabels → {"Name"}, EdgeLabelStyle → {Bold},
  VertexCoordinates → VCL, VertexLabels → Vll, VertexSize → 0.25]
```

Out[88]=



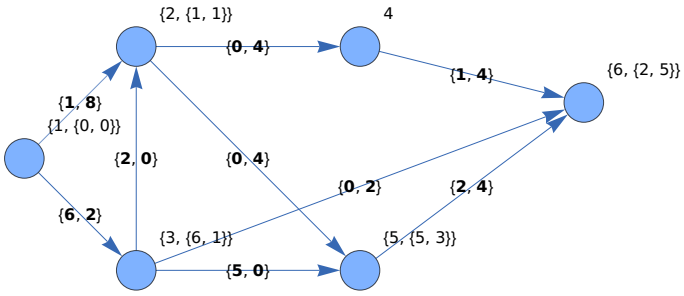
In[93]:=
```
N41 = {{1, {0, 0}}, {2, {1, 1}}, {3, {6, 1}}};
```

In[98]:=
```
N42 = Flow[N41, Cl4]
Vll4 = Vld[N42];
G4a = Graph[Lel4, EdgeLabelStyle → {Bold},
  VertexCoordinates → VCL, VertexLabels → Vll4, VertexSize → 0.25]
N41 = N42;
```

Out[98]=

{{1, {0, 0}}, {2, {1, 1}}, {3, {6, 1}}, {5, {5, 3}}, {6, {2, 5}}}

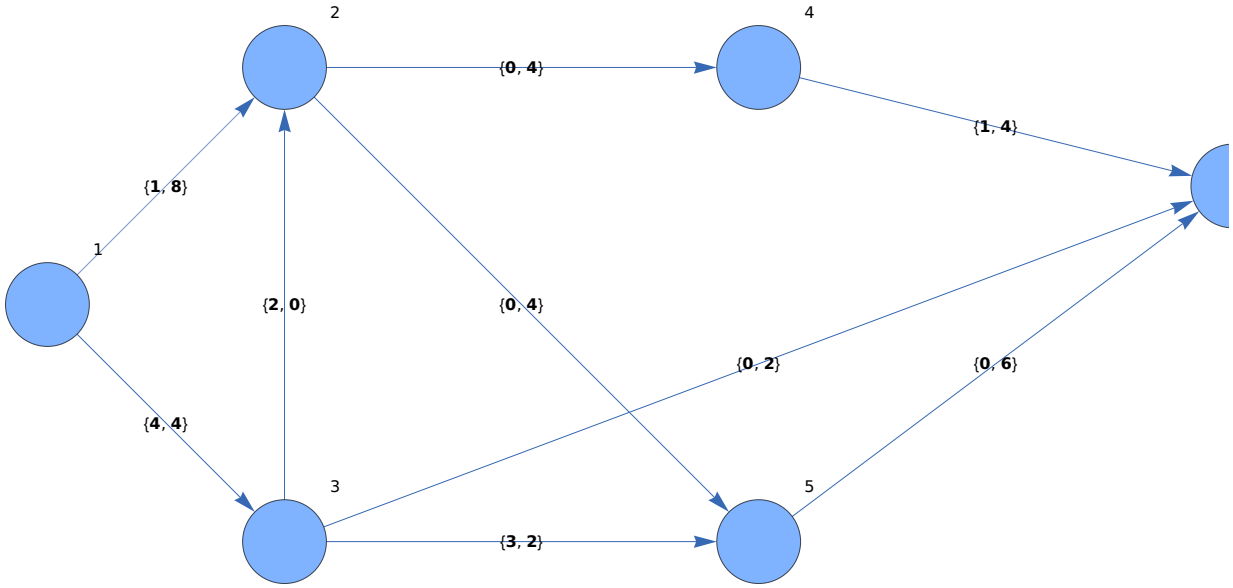
Out[100]=



In[102]:=

```
Fp4 = {6, 5, 3, 1};
Cl5 = Aug[Fp4, Cl4, 2];
Lel5 = Table[
Labeled[El[[i]], Cl5[[i]]], {i, 1, Length[El]}];
Graph[Lel5, VertexLabels → {"Name"}, EdgeLabelStyle → {Bold},
 VertexCoordinates → VCL, VertexLabels → Vll, VertexSize → 0.25]
```

Out[105]=



In[106]:=

```
N51 = {{1, {0, 0}}, {2, {1, 1}}, {3, {4, 1}}};
```

In[111]:=

```
N52 = Flow[N51, Cl5]
Vll5 = Vld[N52];
Graph[Lel5, EdgeLabelStyle → {Bold},
 VertexCoordinates → VCL, VertexLabels → Vll5, VertexSize → 0.25]
N51 = N52;
```

Out[111]=

```
{{1, {0, 0}}, {2, {1, 1}}, {3, {4, 1}}, {5, {3, 3}}}
```

Out[113]=