

$$x_{ri} + \sum_{j \in N} t_{ij} x_j = x_{Bi}$$

$$\text{Max: } x_1 + x_2$$

$$\text{subj. to: } 2x_1 + 3x_2 \leq 12$$

$$2x_1 + x_2 \leq 6$$

~>

$$2x_1 + 3x_2 + x_3 = 12 \Leftrightarrow x_3 = 12 - 2x_1 - 3x_2$$

$$2x_1 + x_2 + x_4 = 6 \Leftrightarrow x_4 = 6 - 2x_1 - x_2$$

$$x_1, x_2 \geq 0$$

x_1 integer, x_2 real.

$$x_2 \begin{pmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 3 \\ 1 & 0 & -\frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{2} \\ 0 & 0 & -\frac{3}{4} & -\frac{3}{4} & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{9}{2} \end{pmatrix}$$

$$\rightsquigarrow x_1 + \left(-\frac{1}{4}\right)x_3 + \frac{3}{4}x_4 = \frac{3}{2}$$

↕

$$-\frac{1}{4}x_3 + \frac{3}{4}x_4 = \frac{3}{2} - x_1$$

$$t_{23} \left(-\frac{1}{4}\right)x_3$$

$$f = \frac{1}{2} + (1 - x_1)$$

$$\frac{1}{f-1}$$

Case 1: $1 - x_1 < 0 \Rightarrow 1 - x_1 \leq -1$

$$t_{23} x_3 \leq f - 1$$

$$\frac{-\frac{1}{4}}{\frac{1}{2}}$$

$$\rightsquigarrow \frac{1}{f-1} t_{23} x_3 \geq 1 \rightarrow \frac{f}{f-1} t_{23} x_3 \geq f$$

$$\frac{1}{4} = \frac{\frac{1}{2}}{\frac{1}{2}-1} \cdot \left(-\frac{1}{4}\right) = \frac{f}{f-1} t_{23} x_3 + \frac{3}{4} x_4$$

$$d_{ij} = \begin{cases} t_{ij} & t_{ij} \geq 0, & x_j \text{ real} \\ \frac{f_i}{f_i - 1} t_{ij}, & t_{ij} < 0 & x_j \text{ real} \\ g_{ij} & g_{ij} \leq f_i & x_j \text{ integral} \\ \frac{f_i}{f_i - 1} (g_{ij} - 1), & g_{ij} > f_i & x_j \text{ integer} \end{cases}$$

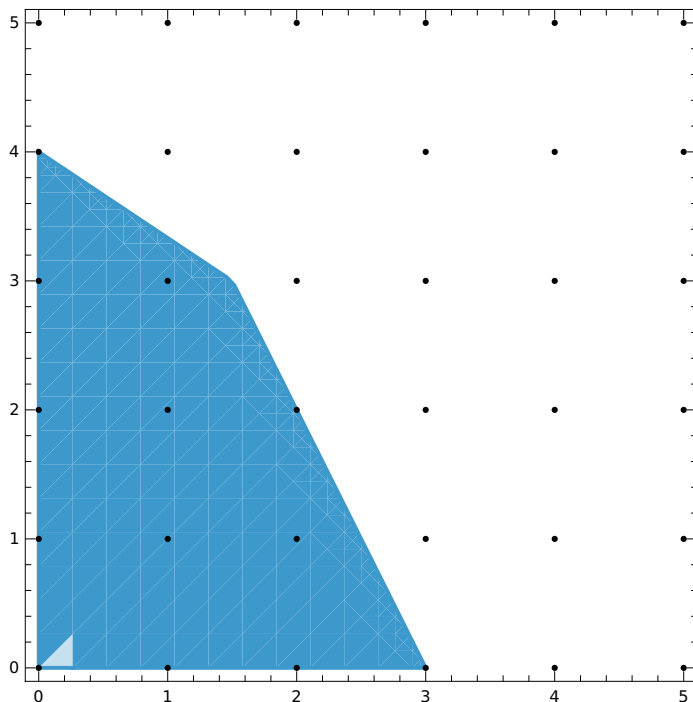
$$- \sum_{j \in N} d_{ij} x_j + u_i = -f_i$$

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In[81]:= T = 5; G1 = RegionPlot[2 x + 3 y ≤ 12 && 2 x + y ≤ 6, {x, 0, T}, {y, 0, T}];
G2 = ListPlot[Table[{i, j}, {i, 0, 10}, {j, 0, 10}],
  PlotStyle → Black, PlotStyle → {PointSize[Medium]}];
G3 = ContourPlot[2 x + 3 y == 12, {x, 0, T}, {y, 0, T}];
G4 = ContourPlot[2 x + y == 6, {x, 0, T}, {y, 0, T}];
G12 = Show[G1, G2]

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Out[85]=



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In[86]:= Pivot[A_, k_, l_] := Module[{B = A},
  m = Dimensions[A][[1]]; n = Dimensions[A][[2]];
  For[i = 1, i ≤ m, i++,
    If[i == k,
      B[[i]] = A[[i]] / A[[k, l]],
      B[[i]] = -A[[i, l]] / A[[k, l]] * A[[k]] + A[[i]]
    ];
  B
]

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In[2]:= MatrixForm[A1 = {{2, 3, 1, 0, 12}, {2, 1, 0, 1, 6}, {-1, -1, 0, 0, 0}}]

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Out[2]//MatrixForm=

$$\begin{pmatrix} 2 & 3 & 1 & 0 & 12 \\ 2 & 1 & 0 & 1 & 6 \\ -1 & -1 & 0 & 0 & 0 \end{pmatrix}$$

In[87]:= **MatrixForm[A2 = Pivot[A1, 2, 1]]**

Out[87]//MatrixForm=

$$\begin{pmatrix} 0 & 2 & 1 & -1 & 6 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 3 \\ 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 3 \end{pmatrix}$$

In[89]:= **MatrixForm[A3 = Pivot[A2, 1, 2]]**

Out[89]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 3 \\ 1 & 0 & -\frac{1}{4} & \frac{3}{4} & \frac{3}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{9}{2} \end{pmatrix}$$

In[90]:= **MatrixForm[A4a = Transpose[Insert[Transpose[A3], {0, 0, 0}, 5]]];**
(*both x1,x2 required integral, unrefined*)

(*unrefined*)

MatrixForm[A4 = Insert[A4a, {0, 0, -3/4, -3/4, 1, -1/2}, 3]]

(*both x1,x2 required integral, refined*)

MatrixForm[A4b = Insert[A4a, {0, 0, -1/4, -1/4, 1, -1/2}, 3]]

(*only x1 required integral*)

MatrixForm[A4c = Insert[A4a, {0, 0, -1/4, -3/4, 1, -1/2}, 3]]

Out[91]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 3 \\ 1 & 0 & -\frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{2} \\ 0 & 0 & -\frac{3}{4} & -\frac{3}{4} & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{9}{2} \end{pmatrix}$$

Out[92]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 3 \\ 1 & 0 & -\frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{2} \\ 0 & 0 & -\frac{1}{4} & -\frac{1}{4} & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{9}{2} \end{pmatrix}$$

Out[93]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 3 \\ 1 & 0 & -\frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{2} \\ 0 & 0 & -\frac{1}{4} & -\frac{3}{4} & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{9}{2} \end{pmatrix}$$

In[230]=

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(*Cutting plane 1*)
X3[x1_, x2_] := 12 - 2 x1 - 3 x2;
X4[x1_, x2_] := 6 - 2 x1 - x2;
cp1[x3_, x4_] := -1/2 + 3/4 x3 + 3/4 x4;
cp1b[x3_, x4_] := -1/2 + 1/4 x3 + 1/4 x4;
cp1c[x3_, x4_] := -1/2 + 1/4 x3 + 3/4 x4;
f1[x1_, x2_] := cp1[X3[x1, x2], X4[x1, x2]];
f1b[x1_, x2_] := cp1b[X3[x1, x2], X4[x1, x2]];
f1c[x1_, x2_] := cp1c[X3[x1, x2], X4[x1, x2]];
(*u1=f1[x1,x2]*)
Simplify[f1[x, y]]
Simplify[f1b[x, y]]
Simplify[f1c[x, y]]
Cut1 = RegionPlot[f1[x, y] ≥ 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → Opacity[0.1]];
Cut1b = RegionPlot[f1b[x, y] ≥ 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → Opacity[0.1]];
Cut1c = RegionPlot[f1c[x, y] ≥ 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → Opacity[0.1]];
(*both x1, x2 integral, unrefined*)
G3 = Show[G12, Cut1]
(*both x1,x2 integral, refined*)
G3b = Show[G12, Cut1b]
(*only x1 assumed integral*)
G3c = Show[G12, Cut1c]

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Out[238]=

$$13 - 3x - 3y$$

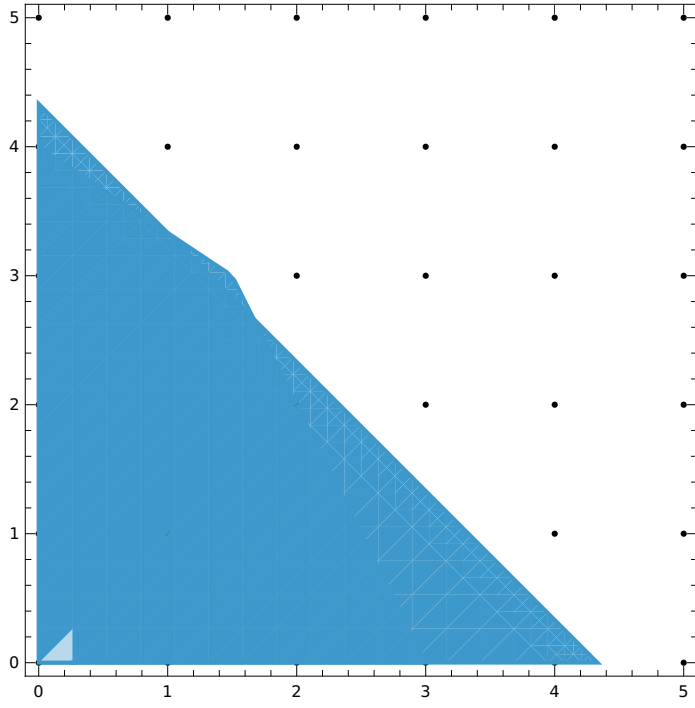
Out[239]=

$$4 - x - y$$

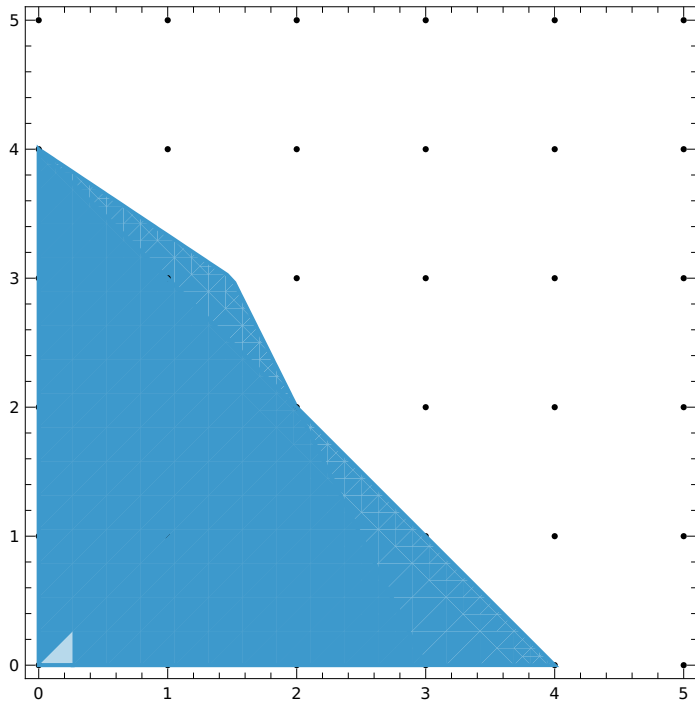
Out[240]=

$$7 - 2x - \frac{3y}{2}$$

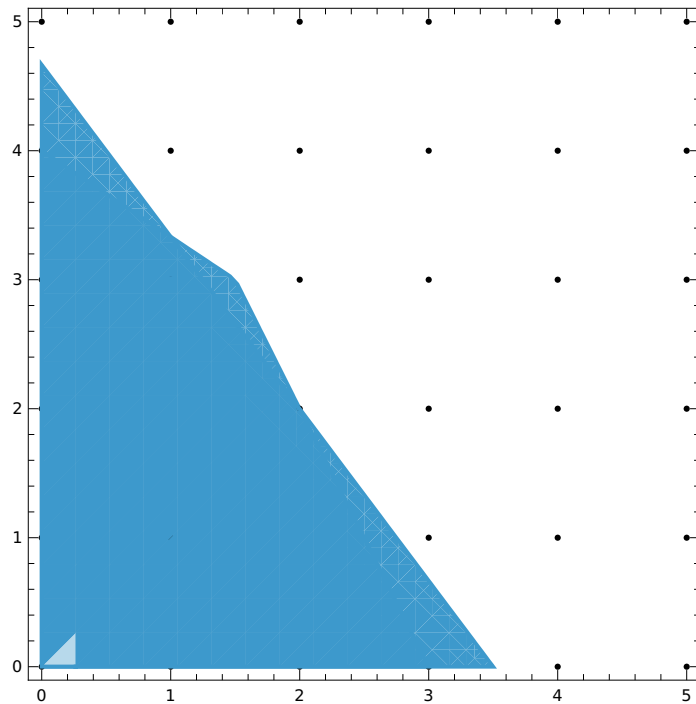
Out[244]=



Out[245]=



Out[246]=



In[227]=

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MatrixForm[A5 = Pivot[A4, 3, 3]]
MatrixForm[A5b = Pivot[A4b, 3, 3]]
MatrixForm[A5c = Pivot[A4c, 3, 4]]

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Out[227]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & -1 & \frac{2}{3} & \frac{8}{3} \\ 1 & 0 & 0 & 1 & -\frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 1 & 1 & -\frac{4}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{13}{3} \end{pmatrix}$$

Out[228]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & -1 & 2 & 2 \\ 1 & 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1 & -4 & 2 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{pmatrix}$$

Out[229]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & \frac{2}{3} & 0 & -\frac{2}{3} & \frac{10}{3} \\ 1 & 0 & -\frac{1}{2} & 0 & 1 & 1 \\ 0 & 0 & \frac{1}{3} & 1 & -\frac{4}{3} & \frac{2}{3} \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{13}{3} \end{pmatrix}$$

In[225]:=

(*x1, x2 required integral, unrefined*)

MatrixForm[A6a = Transpose[Insert[Transpose[A5], {0, 0, 0, 0}, 6]]];

MatrixForm[A6 = Insert[A6a, {0, 0, 0, 0, -2/3, 1, -2/3}, 4]]

Out[226]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & -1 & \frac{2}{3} & 0 & \frac{8}{3} \\ 1 & 0 & 0 & 1 & -\frac{1}{3} & 0 & \frac{5}{3} \\ 0 & 0 & 1 & 1 & -\frac{4}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & -\frac{2}{3} & 1 & -\frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{13}{3} \end{pmatrix}$$

In[247]:=

(*Cutting plane 2*)

cp2[u1_] := -2/3 + 2/3 u1;

f2[x1_, x2_] := cp2[f1[x1, x2]];

(*u2=f2[x1,x2]*)

Simplify[f2[x, y]]

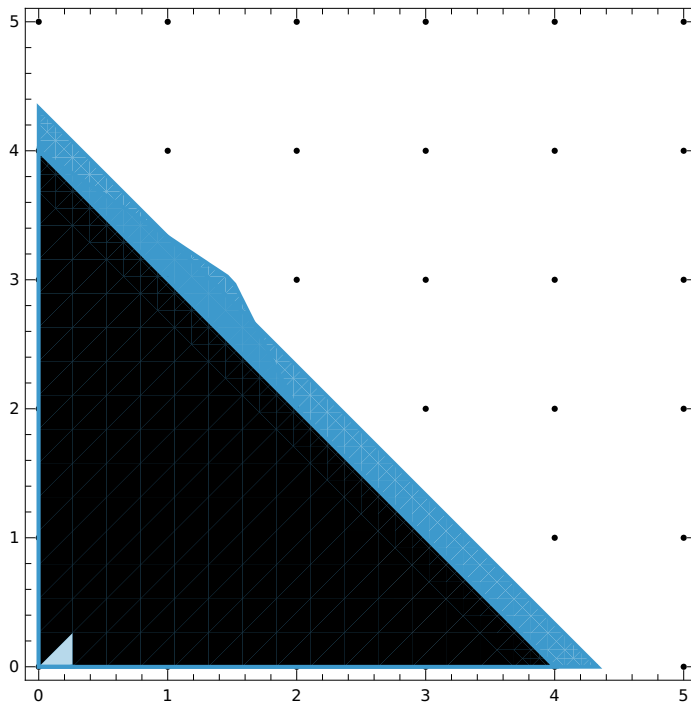
Cut2 = RegionPlot[f2[x, y] ≥ 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → Opacity[0]];

Show[G3, Cut2]

Out[249]=

-2 (-4 + x + y)

Out[251]=



In[252]:=

MatrixForm[A7 = Pivot[A6, 4, 5]]

Out[252]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & -1 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 & 0 & -\frac{1}{2} & 2 \\ 0 & 0 & 1 & 1 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 & 1 & -\frac{3}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 4 \end{pmatrix}$$