

Maximize $Z = x_1 + x_2$

subject to $2x_1 + 3x_2 \leq 12 \Leftrightarrow 2x_1 + 3x_2 + x_3 = 12$

$2x_1 + x_2 \leq 6 \Leftrightarrow 2x_1 + x_2 + x_4 = 6$

$x_1 \geq 0, x_2 \geq 0, \text{integers}$

	x_1	x_2	x_3	x_4	
x_3	2	3	1	0	12
x_4	2	1	0	1	6
	-1	-1	0	0	0

$x_3 = 12 - 2x_1 - 3x_2$ integers

$x_4 = 6 - 2x_1 - x_2$

$(x_1, x_2, x_3, x_4) = (\frac{3}{2}, 3, 0, 0)$

C_B		x_1	x_2	x_3	x_4	x_B
-1	x_2	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	3
-1	x_1	1	0	$-\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{2}$
		0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{9}{2}$

$$1 \cdot x_1 + (-1) x_3 + 0 \cdot x_4$$

$$1 \cdot x_1 + (-1) x_3 + 0 \cdot x_4 + \left(\frac{3}{4} x_3 + \frac{3}{4} x_4\right)$$

$$1 \cdot x_1 - \frac{1}{4} x_3 + \frac{3}{4} x_4 = \frac{3}{2}$$

$$(-1) + \frac{3}{4} \quad 0 + \frac{3}{4} \quad 1 + \frac{1}{2}$$

$$\left[-\frac{1}{4}\right] + \frac{3}{4} \quad \left[\frac{3}{4}\right] + \frac{3}{4}$$

$$\frac{3}{4} x_3 + \frac{3}{4} x_4 = \frac{1}{2} + (-1 \cdot x_1 - (-1) x_3 - 0 \cdot x_4)$$

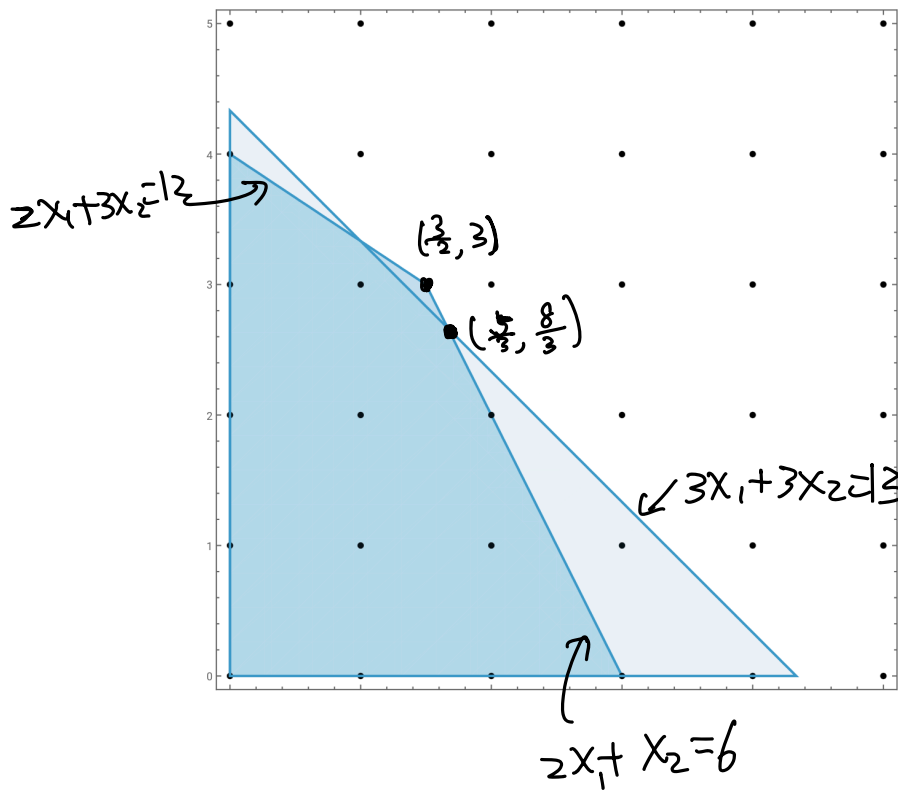
$$\frac{3}{4} x_3 + \frac{3}{4} x_4 \geq \frac{1}{2}$$

$$\Leftrightarrow \frac{27}{2} - 3x_1 - 3x_2 \geq \frac{1}{2}$$

$$\frac{3}{4} (12 - 2x_1 - 3x_2) + \frac{3}{4} (6 - 2x_1 - x_2)$$

$$3x_1 + 3x_2 \leq 13$$

$$9 - \frac{3}{2} x_1 - \frac{9}{4} x_2 + \frac{9}{2} - \frac{3}{2} x_1 - \frac{3}{4} x_2$$



$$\frac{3}{4}x_3 + \frac{3}{4}x_4 \geq \frac{1}{2}$$

$$-\frac{3}{4}x_3 - \frac{3}{4}x_4 + x_5 = -\frac{1}{2}$$

$$x_5 \geq 0$$

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 3 \\ 1 & 0 & -\frac{1}{4} & \frac{3}{4} & \frac{3}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{9}{2} \end{pmatrix}$$

		-1	-1	0	0	0	
		x_1	x_2	x_3	x_4	x_5	
-1	x_2	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	3
-1	x_1	1	0	$-\frac{1}{4}$	$\frac{3}{4}$	0	$\frac{3}{2}$
0	x_5	0	0	$-\frac{3}{4}$	$-\frac{3}{4}$	1	$-\frac{1}{2}$
				$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{9}{2}$

		x_3	x_4	x_5	
x_2	0	1	0	-1	$\frac{2}{3}$ $\frac{8}{3}$
x_1	1	0	0	1	$\frac{1}{3}$ $\frac{5}{3}$
x_3	0	0	1	1	$\frac{4}{3}$ $\frac{2}{3}$
	0	0	0	0	$\frac{1}{3}$ $\frac{13}{3}$

$$(x_1, x_2, x_3, x_4, x_5) = (\frac{5}{3}, \frac{8}{3}, 0, 0, 0)$$

$$(x_1, x_2) = (\frac{5}{3}, \frac{8}{3})$$

\uparrow
 \uparrow
 $\frac{1}{3}$ $\frac{2}{3}$

\Rightarrow

$$x_1 + x_4 - \frac{1}{3}x_5 = \frac{5}{3} \quad \leadsto \quad \frac{2}{3}x_5 = \frac{2}{3} + \underbrace{1 - (x_1 + x_4 - 1 \cdot x_5)}_0$$

$$x_1 + x_4 - 1 \cdot x_5 \leq 1$$

$$\Rightarrow \frac{2}{3}x_5 \geq \frac{2}{3}$$

$$x_{ri} + \sum_{j \in N} t_{ij} x_j = \frac{x_{Bi}}{||} \Rightarrow \sum_{j \in N} g_{ij} x_j \geq f_i$$

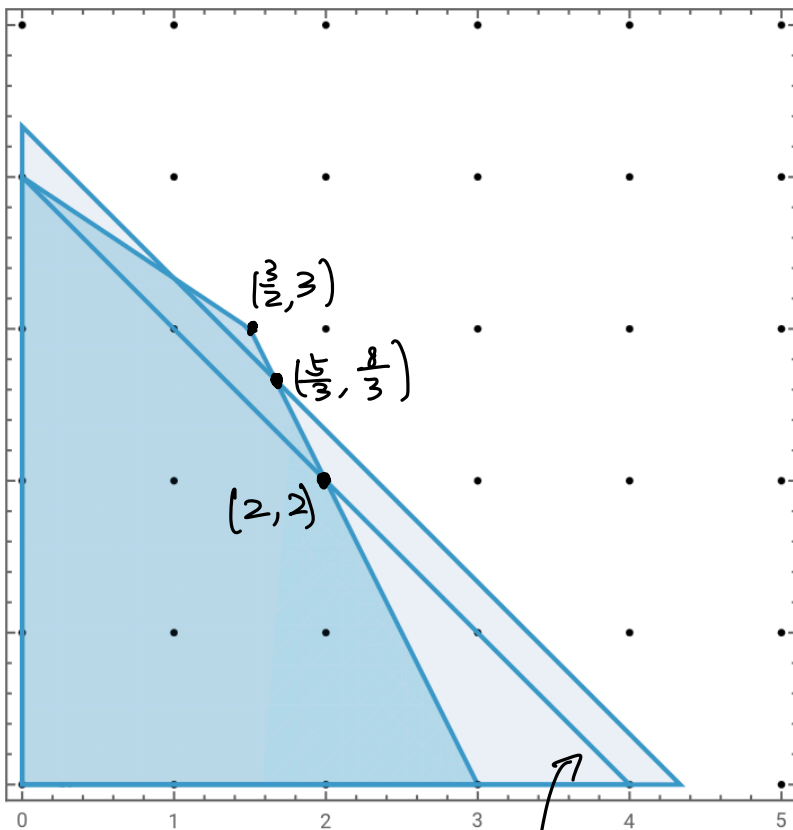
$$[t_{ij}] + g_{ij} \quad [x_{Bi}] + f_i$$

$$u_i = -f_i + \sum_{j \in N} g_{ij} x_j$$

$$-\frac{2}{3}x_5 + x_6 = -\frac{2}{3}$$

0	1	0	-1	$\frac{2}{3}$	$\frac{8}{3}$
1	0	0	1	$-\frac{1}{3}$	$\frac{5}{3}$
0	0	1	1	$-\frac{4}{3}$	$\frac{2}{3}$
0	0	0	0	$\frac{1}{3}$	$\frac{13}{3}$

	x_1	x_2	x_3	x_4	x_5	x_6	x_B
x_2	0	1	0	-1	$\frac{2}{3}$	0	$\frac{8}{3}$
x_1	1	0	0	1	$-\frac{1}{3}$	0	$\frac{5}{3}$
x_3	0	0	1	1	$-\frac{4}{3}$	0	$\frac{2}{3}$
x_6	0	0	0	0	$-\frac{1}{3}$	1	$\frac{1}{3}$
	0	0	0	0	$\frac{1}{3}$	0	$\frac{13}{3}$



$$x_1 + x_2 = 4$$

$$\Leftrightarrow \frac{2}{3}x_5 \geq \frac{2}{3}$$

$$\begin{aligned} &\Downarrow \\ &x_5 \geq 1 \\ &\uparrow \\ &13 - 3x_1 - 3x_2 \end{aligned}$$

$$\Updownarrow \\ x_1 + x_2 \leq 4$$

$$-\frac{3}{4}x_3 - \frac{3}{4}x_4 + x_5 = -\frac{1}{2}$$

$$\begin{aligned} x_3 &= 12 - 2x_1 - 3x_2 \\ x_4 &= 6 - 2x_1 - x_2 \end{aligned}$$

$$x_5 = -\frac{1}{2} + \frac{3}{4}x_3 + \frac{3}{4}x_4$$

$$= -\frac{1}{2} + \frac{3}{4}(12 - 2x_1 - 3x_2) + \frac{3}{4}(6 - 2x_1 - x_2)$$

$$= -\frac{1}{2} + 9 - \frac{3}{2}x_1 - \frac{9}{4}x_2 + \frac{9}{2} - \frac{3}{2}x_1 - \frac{3}{4}x_2$$

$$= 13 - 3x_1 - 3x_2$$

$$x_{B_i} = \lfloor x_{B_i} \rfloor + f_i, \quad 0 < f_i < 1$$

$$x_{r_i} + \sum_{j \in N} t_{ij} x_j = x_{B_i}$$

$$t_{ij} = \lfloor t_{ij} \rfloor + g_{ij}$$

$$\Rightarrow \sum_{j \in N} g_{ij} x_j \geq f_i$$

$$\Uparrow$$

$$\boxed{u_i = -f_i + \sum_{j \in N} g_{ij} x_j \geq 0}$$

$$-\sum_{j \in N} g_{ij} x_j + u_i = -f_i$$

		u_i	
		0	
	$-g_{ij}$	1	$-f_i$
		0	

$$x_{Bi} = \lfloor x_{Bi} \rfloor + f_i$$

$$x_{r_i} + \underbrace{\sum_{j \in N} t_{ij} x_j}_{\parallel} = x_{Bi}$$

$$\underbrace{\lfloor t_{il} \rfloor}_{\parallel} + g_{il}$$

$$\sum_{j \in N^-} t_{ij} x_j + \sum_{k \in N^+} t_{ik} x_k + \sum_l \underbrace{t_{il}}_{x_l \text{ integral}} x_l$$

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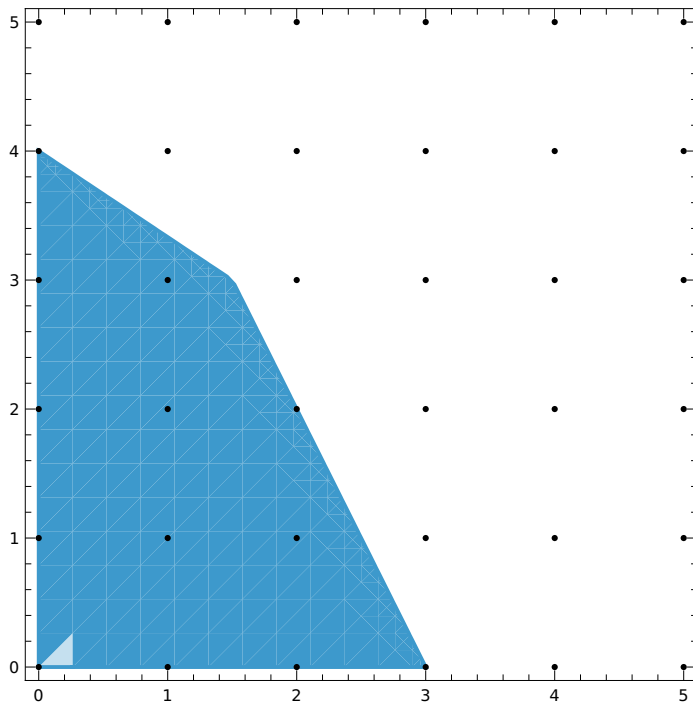
$$\frac{f_i}{f_i - 1} \sum_{j \in N^-} t_{ij} x_j + \sum_{k \in N^+} t_{ik} x_k + \sum_l g_{il} x_l \geq f_i$$

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In[43]:= G1 = RegionPlot[2 x + 3 y ≤ 12 && 2 x + y ≤ 6, {x, 0, 5}, {y, 0, 5}];
G2 = ListPlot[Table[{i, j}, {i, 0, 10}, {j, 0, 10}],
  PlotStyle → Black, PlotStyle → {PointSize[Medium]}];
G12 = Show[G1, G2]

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Out[45]=



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In[5]:= Pivot[A_, k_, l_] := Module[{B = A},
  m = Dimensions[A][[1]]; n = Dimensions[A][[2]];
  For[i = 1, i ≤ m, i++,
    If[i == k,
      B[[i]] = A[[i]] / A[[k, l]],
      B[[i]] = -A[[i, l]] / A[[k, l]] * A[[k]] + A[[i]]
    ];
  B
]

```

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In[4]:= MatrixForm[A1 = {{2, 3, 1, 0, 12}, {2, 1, 0, 1, 6}, {-1, -1, 0, 0, 0}}]

```

Out[4]//MatrixForm=

$$\begin{pmatrix} 2 & 3 & 1 & 0 & 12 \\ 2 & 1 & 0 & 1 & 6 \\ -1 & -1 & 0 & 0 & 0 \end{pmatrix}$$

In[6]:= **MatrixForm[A2 = Pivot[A1, 2, 1]]**

Out[6]//MatrixForm=

$$\begin{pmatrix} 0 & 2 & 1 & -1 & 6 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 3 \\ 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 3 \end{pmatrix}$$

In[7]:= **MatrixForm[A3 = Pivot[A2, 1, 2]]**

Out[7]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 3 \\ 1 & 0 & -\frac{1}{4} & \frac{3}{4} & \frac{3}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{9}{2} \end{pmatrix}$$

In[8]:= **MatrixForm[A4a = Transpose[Insert[Transpose[A3], {0, 0, 0}, 5]]];**
MatrixForm[A4 = Insert[A4a, {0, 0, -3/4, -3/4, 1, -1/2}, 3]]

Out[9]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 3 \\ 1 & 0 & -\frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{2} \\ 0 & 0 & -\frac{3}{4} & -\frac{3}{4} & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{9}{2} \end{pmatrix}$$

```

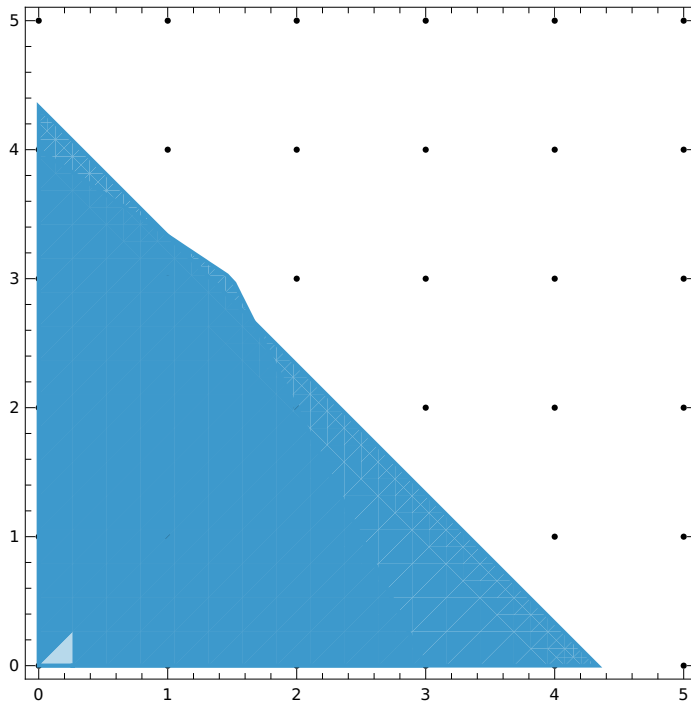
In[46]:= (*Cutting plane 1*)
X3[x1_, x2_] := 12 - 2 x1 - 3 x2;
X4[x1_, x2_] := 6 - 2 x1 - x2;
cp1[x3_, x4_] := -1/2 + 3/4 x3 + 3/4 x4;
f1[x1_, x2_] := cp1[X3[x1, x2], X4[x1, x2]];
(*u1=f1[x1,x2]*)
Simplify[f1[x, y]]
Cut1 = RegionPlot[f1[x, y] ≥ 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → Opacity[0.1]];
G3 = Show[G12, Cut1]

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Out[50]=

$13 - 3x - 3y$

Out[52]=



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In[17]:= MatrixForm[A5 = Pivot[A4, 3, 3]]

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Out[17]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & -1 & \frac{2}{3} & \frac{8}{3} \\ 1 & 0 & 0 & 1 & -\frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 1 & 1 & -\frac{4}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{13}{3} \end{pmatrix}$$

```
In[18]:= MatrixForm[A6a = Transpose[Insert[Transpose[A5], {0, 0, 0, 0}, 6]]];  
MatrixForm[A6 = Insert[A6a, {0, 0, 0, 0, -2/3, 1, -2/3}, 4]]
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & -1 & \frac{2}{3} & 0 & \frac{8}{3} \\ 1 & 0 & 0 & 1 & -\frac{1}{3} & 0 & \frac{5}{3} \\ 0 & 0 & 1 & 1 & -\frac{4}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & -\frac{2}{3} & 1 & -\frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{13}{3} \end{pmatrix}$$

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In[32]:= MatrixForm[A7 = Pivot[A6, 4, 5]]
```

Out[32]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & -1 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 & 0 & -\frac{1}{2} & 2 \\ 0 & 0 & 1 & 1 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 & 1 & -\frac{3}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 4 \end{pmatrix}$$

```
In[58]:= (*Cutting plane 2*)
cp2[u1_] := -2/3 + 2/3 u1;
f2[x1_, x2_] := cp2[f1[x1, x2]];
(*u2=f2[x1,x2]*)
Simplify[f2[x, y]]
Cut2 = RegionPlot[f2[x, y] ≥ 0, {x, 0, 5}, {y, 0, 5}, PlotStyle → Opacity[0]];
Show[G3, Cut2]
```

Out[60]=

$-2(-4 + x + y)$

Out[62]=

