Minimize $z = 8x_1 + 6x_2 + 11x_3$

subject to $5x_1 + x_2 + 3x_3 \le 4$

$-5x_1 - x_2 - 3x_3 \le -2 \iff 5x_1 + x_2 + 3x_3 \ge 2$

$2x_1 + 4x_2 + 7x_3 \le 5$

$-2x_1 - 4x_2 - 7x_3 \le -3 \iff 2x_1 + 4x_2 + 7x_3 \ge 3$

$x_1 + x_2 + x_3 \le 1 \iff x_1 + x_2 + x_3 = 1$

$-x_1 - x_2 - x_3 \le -1$

$x_1, x_2, x_3 \ge 0$

$\iff$

$-\text{Max.}\ -8x_1 - 6x_2 - 11x_3$

$5x_1 + x_2 + 3x_3 + x_4 = 4$

$-5x_1 - x_2 - 3x_3 + x_5 = -2$

$2x_1 + 4x_2 + 7x_3 + x_6 = 5$

$-2x_1 - 4x_2 - 7x_3 + x_7 = -3$

$x_1 + x_2 + x_3 + x_8 = 1$

$-x_1 - x_2 - x_3 + x_9 = -1$.

$x_i \ge 0,\ i = 1, \cdots, 9$.

		-8 x_1	-6 x_2	-11 x_3	0 x_4	0 x_5	0 x_6	0 x_7	0 x_8	0 x_9	
0	x_4	5	1	3	1	0	0	0	0	0	4
0	x_5	-5	-1	-3	0	1	0	0	0	0	-2
0	x_6	2	4	7	0	0	1	0	0	0	5
← 0	x_7	-2	-4	-7	0	0	0	1	0	0	-3 ←
0	x_8	1	1	1	0	0	0	0	1	0	1
0	x_9	-1	-1	-1	0	0	0	0	0	1	-1
		8	6	11	0	0	0	0	0	0	0

↑ ↑ ↑

$\frac{8}{2}$ $\frac{6}{4}$ $\frac{11}{7}$

$4 > \frac{3}{2} < \frac{11}{7}$

Dual Simplex Method →

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	
x_4	0	0	0	1	1	0	0	0	0	2
x_7	0	0	-4	0	$\frac{1}{2}$	0	1	0	$-\frac{9}{2}$	$\frac{1}{2}$
x_6	0	0	4	0	$-\frac{1}{2}$	1	0	0	$\frac{9}{2}$	$\frac{3}{2}$
x_1	1	0	$\frac{1}{2}$	0	$-\frac{1}{4}$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$
x_8	0	0	0	0	0	0	0	1	1	0
x_2	0	1	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0	0	$-\frac{5}{4}$	$\frac{3}{4}$
	0	0	4	0	$\frac{1}{2}$	0	0	0	$\frac{11}{2}$	$-\frac{13}{2}$

$(x_1, x_2, x_3) = (\frac{1}{4}, \frac{3}{4}, 0)$ ⇐

$-\underset{\text{Min}}{\text{Max}} = -(-\frac{13}{2}) = \frac{13}{2}$

Midterm Problem:

- Maximize $z = 3x_1 - x_2$

  subject to $x_1 + x_2 \geq 1$

  $-x_1 + x_2 \leq 1$

  $2x_1 - x_2 \leq 2$

2-phase method $\rightsquigarrow$ last tableaux. $\Rightarrow$ optimal solution

	x_1	x_2	x_3	x_4	x_5	
x_1	1	0	0	1	1	3
x_2	0	1	0	2	1	4
x_3	0	0	1	3	2	6
	0	0	0	1	2	5

$(x_1, x_2) = (3, 4)$.

Maximum $= 5$.

Now: add one more constraint:

$2x_1 + x_2 \leq 4 \iff 2x_1 + x_2 + x_6 = 4$.

$x_6 \geq 0$

$\rightsquigarrow$ New tableaux

	x_1	x_2	x_3	x_4	x_5	x_6	
x_1	1	0	0	1	1	0	3
x_2	0	1	0	2	1	0	4
x_3	0	0	1	3	2	0	6
x_6	2	1	0	0	0	1	4
	0	0	0	1	2	0	5

$\rightsquigarrow$

	x_1	x_2	x_3	x_4	x_5	x_6	
x_1	1	0	0	1	1	0	3
x_2	0	1	0	2	1	0	4
x_3	0	0	1	3	2	0	6
x_6	0	0	0	-4	-3	1	-6
	0	0	0	1	2	0	5

} Dual Simplex method

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ \\ \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{3}{2} \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 & 0 & -\frac{1}{4} & \frac{3}{4} & \frac{3}{2} \\ 0 & 0 & 0 & 1 & \frac{3}{4} & -\frac{1}{4} & \frac{3}{2} \\ 0 & 0 & 0 & 0 & \frac{5}{4} & \frac{1}{4} & \frac{7}{2} \end{pmatrix}$$

New optimal solution

$(x_1, x_2) = \left(\frac{3}{2}, 1\right)$.

Maximum: $\frac{7}{2}$