

3.3: 8. Minimize  $z = 8x_1 + 6x_2 + 11x_3$   
 subject to  $5x_1 + x_2 + 3x_3 \leq 4$   
 $5x_1 + x_2 + 3x_3 \geq 2$   
 $2x_1 + 4x_2 + 7x_3 \leq 5$   
 $2x_1 + 4x_2 + 7x_3 \geq 3$   
 $x_1 + x_2 + x_3 = 1$   
 $x_1, x_2, x_3 \geq 0$



— Maximize  $-8x_1 - 6x_2 - 11x_3$   
 subject to  $5x_1 + x_2 + 3x_3 + x_4 = 4$   
 $5x_1 + x_2 + 3x_3 - x_5 + y_1 = 2$   
 $2x_1 + 4x_2 + 7x_3 + x_6 = 5$   
 $2x_1 + 4x_2 + 7x_3 - x_7 + y_2 = 3$   
 $x_1 + x_2 + x_3 + y_3 = 1$   
 $x_i \geq 0, y_j \geq 0$

Phase I: Min.  $y_1 + y_2 + y_3 \iff$  Max:  $\ominus y_1 \ominus y_2 - y_3$

		0	0	0	0	0	0	0	-1	-1	-1	
CB		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$y_1$	$y_2$	$y_3$	
0	$x_4$	5	1	3	1	0	0	0	0	0	0	4
-1	$y_1$	5	1	3	0	-1	0	0	1	0	0	2
0	$x_6$	2	4	7	0	0	1	0	0	0	0	5
-1	$y_2$	2	4	7	0	0	0	-1	0	1	0	3
-1	$y_3$	1	1	1	0	0	0	0	0	0	1	1
		-8	-6	-11	0	1	0	1	0	0	0	-6

Phase 2:

- Max:  $-8x_1 - 6x_2 - 11x_3$

$C_B$		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$y_1$	$y_2$	$y_3$	
0	$x_4$	0	0	0	1	1	0	0	-1	0	0	2
-8	$x_1$	1	0	$\frac{1}{2}$	0	$-\frac{1}{4}$	0	0	$\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{1}{4}$
0	$x_6$	0	0	4	0	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{9}{2}$	$\frac{3}{2}$
-6	$x_2$	0	1	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0	$-\frac{1}{4}$	0	$\frac{5}{4}$	$\frac{3}{4}$
0	$x_7$	0	0	-4	0	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	-1	$\frac{9}{2}$	$\frac{1}{2}$
		0	0	4	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$-\frac{11}{2}$	$-\frac{13}{2}$

$-8 \cdot \frac{1}{2} - 6 \cdot \frac{1}{2} - (-11)$

||

$-4 - 3 + 11$

$(-8) \cdot (-\frac{1}{4}) - 6 \cdot \frac{1}{4}$

||

$2 - \frac{3}{2} = \frac{1}{2}$

$-8 \cdot \frac{1}{4} - 6 \cdot \frac{3}{4}$

||

$-2 - \frac{9}{2} = -\frac{13}{2}$

$\Rightarrow$  optimal solution  $(x_1, x_2, x_3) = (\frac{1}{4}, \frac{3}{4}, 0)$ .

Min = -Max =  $-(-\frac{13}{2}) = \frac{13}{2}$

$$B = (A_{x_4} A_{x_1} A_{x_6} A_{x_2} A_{x_7})$$

	$y_1$	$y_2$	$y_3$
$x_4$	0	0	0
$x_1$	1	0	$\frac{1}{2}$
$x_6$	0	0	4
$x_2$	0	1	$\frac{1}{2}$
$x_7$	0	0	-4
	0	0	4

Minimize  $z = 8x_1 + 6x_2 + 11x_3$   
 subject to  
 $5x_1 + x_2 + 3x_3 \leq 4$   
 $5x_1 + x_2 + 3x_3 \geq 2$   
 $2x_1 + 4x_2 + 7x_3 \leq 5$   
 $2x_1 + 4x_2 + 7x_3 \geq 3$   
 $x_1 + x_2 + x_3 = 1$   
 $x_1, x_2, x_3 \geq 0$

$(-)$  Max.  $(-8)x_1 - 6x_2 - 11x_3$   
 $5x_1 + x_2 + 3x_3 \leq 4$   
 $-5x_1 - x_2 - 3x_3 \leq -2$   
 $2x_1 + 4x_2 + 7x_3 \leq 5$   
 $-2x_1 - 4x_2 - 7x_3 \leq -3$   
 $1x_1 + x_2 + x_3 = 1$   
 $1x_1, x_2, x_3 \geq 0$

Dual Problem:  $(-)$  Min.  $4w_1 - 2w_2 + 5w_3 - 3w_4 + 1w_5$

subject to  $5w_1 - 5w_2 + 2w_3 - 2w_4 + w_5 \geq -8$

Max:  $-4w_1 + 2w_2 - 5w_3 + 3w_4 - w_5$   $1 \cdot w_1 - 1 \cdot w_2 + 4w_3 - 4w_4 + w_5 \geq -6$

$3 \cdot w_1 - 3 \cdot w_2 + 7 \cdot w_3 - 7 \cdot w_4 + 1 \cdot w_5 \geq -11$

$w_1 \geq 0, w_2 \geq 0, w_3 \geq 0, w_4 \geq 0, w_5$  unrestricted

Optimal solution to the primal problem:

$$z = C_B^T \cdot (B^{-1} \cdot b) = b^T \cdot \underbrace{(B^{-1})^T \cdot C_B}_{\hat{w}}$$

Optimal solution to the dual problem:  $\hat{w}^T = C_B^T \cdot B^{-1}$

$C_B^T \cdot B^{-1} = (0, \frac{1}{2}, 0, 0, -\frac{11}{2})$

	$C_B$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$y_1$	$y_2$	$y_3$	
$-8 -6 -11$	$C_B$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$y_1$	$y_2$	$y_3$	
$0$	$x_4$	0	0	0	1	1	0	0	-1	0	0	2
$-6$	$x_1$	1	0	$\frac{1}{2}$	0	$-\frac{1}{4}$	0	0	$\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{1}{4}$
$0$	$x_6$	0	0	4	0	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{9}{2}$	$\frac{3}{2}$
$-6$	$x_2$	0	1	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0	$-\frac{1}{4}$	0	$\frac{5}{4}$	$\frac{3}{4}$
$0$	$x_7$	0	0	-4	0	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	-1	$\frac{9}{2}$	$\frac{1}{2}$
		0	0	4	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$-\frac{11}{2}$	$-\frac{13}{2}$

←  $A'$

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & -\frac{1}{4} \\ 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & -\frac{1}{2} & 0 & -1 & \frac{1}{2} \end{pmatrix}$$

$$B^{-1} = (A'_{x_4}, A'_{y_1}, A'_{x_6}, A'_{y_2}, A'_{y_3})$$

$$B = \begin{pmatrix} 1 & 5 & 0 & 1 & 0 \\ 0 & 5 & 0 & 1 & 0 \\ 0 & 2 & 1 & 4 & 0 \\ 0 & 2 & 0 & 4 & -1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$\uparrow \uparrow \uparrow \uparrow \uparrow$   
 $x_4 \ x_1 \ x_6 \ x_2 \ x_7$

$C_B$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$y_1$	$y_2$	$y_3$	
$0$	$x_4$	5	1	3	1	0	0	0	0	0	4
$-1$	$y_1$	5	1	3	0	-1	0	0	1	0	2
$0$	$x_6$	2	4	7	0	0	1	0	0	0	5
$-1$	$y_2$	2	4	7	0	0	0	-1	0	1	3
$-1$	$y_3$	1	1	1	0	0	0	0	0	1	1
		-8	-6	-11	0	1	0	1	0	0	-6

$$\hat{w} = (\hat{w}_1, \hat{w}_2, \hat{w}_3, \hat{w}_4, \hat{w}_5)$$

$$C_B^T \cdot B^{-1} = (0, \frac{1}{2}, 0, 0, -\frac{11}{2}) \rightarrow w = (0, \frac{1}{2}, 0, 0, -\frac{11}{2})$$

$$\text{Max: } -4w_1 + 2w_2 - 5w_3 + 3w_4 - w_5 = 2 \cdot \frac{1}{2} + \frac{11}{2} = \frac{13}{2}$$

$$\begin{array}{ll}
 \text{Minimize } z = 8x_1 + 6x_2 + 11x_3 & \text{--- Max } (-8x_1 - 6x_2 - 11x_3) \\
 \text{subject to } 5x_1 + x_2 + 3x_3 \leq 4 & 5x_1 + x_2 + 3x_3 + x_4 = 4 \\
 5x_1 + x_2 + 3x_3 \geq 2 & \Leftrightarrow -5x_1 - x_2 - 3x_3 + x_5 = -2 \\
 2x_1 + 4x_2 + 7x_3 \leq 5 & 2x_1 + 4x_2 + 7x_3 + x_6 = 5 \\
 2x_1 + 4x_2 + 7x_3 \geq 3 & -2x_1 - 4x_2 - 7x_3 + x_7 = -3 \\
 x_1 + x_2 + x_3 = 1 & \Leftrightarrow x_1 + x_2 + x_3 + x_8 = 1 \\
 x_1, x_2, x_3 \geq 0 & -x_1 - x_2 - x_3 + x_9 = -1
 \end{array}$$

Dual Simplex  
Method  $\rightarrow$

		-8	-6	-11	0	0	0	0	0	0	
		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	
0	$x_4$	5	1	3	1	0	0	0	0	0	4
0	$x_5$	-5	-1	-3	0	1	0	0	0	0	-2
0	$x_6$	2	4	7	0	0	1	0	0	0	5
0	$x_7$	-2	-4	-7	0	0	0	1	0	0	-3
0	$x_8$	1	1	1	0	0	0	0	1	0	1
0	$x_9$	-1	-1	-1	0	0	0	0	0	1	-1
		8	6	11	0	0	0	0	0	0	0

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In[2]:= Pivot[A_, k_, l_] := Module[{B = A},
  m = Dimensions[A][[1]]; n = Dimensions[A][[2]];
  For[i = 1, i ≤ m, i++,
    If[i == k,
      B[[i]] = A[[i]] / A[[k, l]],
      B[[i]] = -A[[i, l]] / A[[k, l]] * A[[k]] + A[[i]]
    ]
  ];
  B
]

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(\*3.3:Exercise 8\*)

(\*Phase 1\*)

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In[6]:= MatrixForm[A1 = {{5, 1, 3, 1, 0, 0, 0, 0, 0, 0, 4}, {5, 1, 3, 0, -1, 0, 0, 1, 0, 0, 2},
  {2, 4, 7, 0, 0, 1, 0, 0, 0, 0, 5}, {2, 4, 7, 0, 0, 0, -1, 0, 1, 0, 3},
  {1, 1, 1, 0, 0, 0, 0, 0, 1, 1}, {-8, -6, -11, 0, 1, 0, 1, 0, 0, 0, -6}}]

```

Out[6]//MatrixForm=

$$\begin{pmatrix} 5 & 1 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 5 & 1 & 3 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 2 \\ 2 & 4 & 7 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 5 \\ 2 & 4 & 7 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 3 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & \\ -8 & -6 & -11 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & -6 \end{pmatrix}$$

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In[9]:= MatrixForm[A2 = Pivot[A1, 4, 3]]

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Out[9]//MatrixForm=

$$\begin{pmatrix} \frac{29}{7} & -\frac{5}{7} & 0 & 1 & 0 & 0 & \frac{3}{7} & 0 & -\frac{3}{7} & 0 & \frac{19}{7} \\ \frac{29}{7} & -\frac{5}{7} & 0 & 0 & -1 & 0 & \frac{3}{7} & 1 & -\frac{3}{7} & 0 & \frac{5}{7} \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 2 \\ \frac{2}{7} & \frac{4}{7} & 1 & 0 & 0 & 0 & -\frac{1}{7} & 0 & \frac{1}{7} & 0 & \frac{3}{7} \\ \frac{5}{7} & \frac{3}{7} & 0 & 0 & 0 & 0 & \frac{1}{7} & 0 & -\frac{1}{7} & 1 & \frac{4}{7} \\ -\frac{34}{7} & \frac{2}{7} & 0 & 0 & 1 & 0 & -\frac{4}{7} & 0 & \frac{11}{7} & 0 & -\frac{9}{7} \end{pmatrix}$$

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In[10]:= MatrixForm[A3 = Pivot[A2, 2, 1]]

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Out[10]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 2 \\ 1 & -\frac{5}{29} & 0 & 0 & -\frac{7}{29} & 0 & \frac{3}{29} & \frac{7}{29} & -\frac{3}{29} & 0 & \frac{5}{29} \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 2 \\ 0 & \frac{18}{29} & 1 & 0 & \frac{2}{29} & 0 & -\frac{5}{29} & -\frac{2}{29} & \frac{5}{29} & 0 & \frac{11}{29} \\ 0 & \frac{16}{29} & 0 & 0 & \frac{5}{29} & 0 & \frac{2}{29} & -\frac{5}{29} & -\frac{2}{29} & 1 & \frac{13}{29} \\ 0 & -\frac{16}{29} & 0 & 0 & -\frac{5}{29} & 0 & -\frac{2}{29} & \frac{34}{29} & \frac{31}{29} & 0 & -\frac{13}{29} \end{pmatrix}$$

In[11]:= **MatrixForm[A4 = Pivot[A3, 4, 2]]**

Out[11]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 2 \\ 1 & 0 & \frac{5}{18} & 0 & -\frac{2}{9} & 0 & \frac{1}{18} & \frac{2}{9} & -\frac{1}{18} & 0 & \frac{5}{18} \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & \frac{29}{18} & 0 & \frac{1}{9} & 0 & -\frac{5}{18} & -\frac{1}{9} & \frac{5}{18} & 0 & \frac{11}{18} \\ 0 & 0 & -\frac{8}{9} & 0 & \frac{1}{9} & 0 & \frac{2}{9} & -\frac{1}{9} & -\frac{2}{9} & 1 & \frac{1}{9} \\ 0 & 0 & \frac{8}{9} & 0 & -\frac{1}{9} & 0 & -\frac{2}{9} & \frac{10}{9} & \frac{11}{9} & 0 & -\frac{1}{9} \end{pmatrix}$$

In[13]:= **MatrixForm[A5 = Pivot[A4, 5, 7]]**

Out[13]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 2 \\ 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 4 & 0 & -\frac{1}{2} & 1 & 0 & \frac{1}{2} & 0 & -\frac{9}{2} & \frac{3}{2} \\ 0 & 1 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & -\frac{1}{4} & 0 & \frac{5}{4} & \frac{3}{4} \\ 0 & 0 & -4 & 0 & \frac{1}{2} & 0 & 1 & -\frac{1}{2} & -1 & \frac{9}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

(\*Phase 2\*)

(\*Keep the artificial column to find solution to the dual problem\*)

In[33]:= **B0 = A5; B0[[6]] = {8, 6, 11, 0, 0, 0, 0, 0, 0, 0, 0};**

**MatrixForm[B0]**

Out[34]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 2 \\ 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 4 & 0 & -\frac{1}{2} & 1 & 0 & \frac{1}{2} & 0 & -\frac{9}{2} & \frac{3}{2} \\ 0 & 1 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & -\frac{1}{4} & 0 & \frac{5}{4} & \frac{3}{4} \\ 0 & 0 & -4 & 0 & \frac{1}{2} & 0 & 1 & -\frac{1}{2} & -1 & \frac{9}{2} & \frac{1}{2} \\ 8 & 6 & 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[35]:= **B1 = B0; B1[[6]] = B0[[6]] - 8 \* B0[[2]] - 6 \* B0[[4]];**

**MatrixForm[B1]**

Out[36]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 2 \\ 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 4 & 0 & -\frac{1}{2} & 1 & 0 & \frac{1}{2} & 0 & -\frac{9}{2} & \frac{3}{2} \\ 0 & 1 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & -\frac{1}{4} & 0 & \frac{5}{4} & \frac{3}{4} \\ 0 & 0 & -4 & 0 & \frac{1}{2} & 0 & 1 & -\frac{1}{2} & -1 & \frac{9}{2} & \frac{1}{2} \\ 0 & 0 & 4 & 0 & \frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 & -\frac{11}{2} & -\frac{13}{2} \end{pmatrix}$$

In[37]:= **MatrixForm**[

**B = Transpose**[[{1, 0, 0, 0, 0}, {5, 5, 2, 2, 1}, {0, 0, 1, 0, 0}, {1, 1, 4, 4, 1}, {0, 0, 0, -1, 0}]]

Out[37]//MatrixForm=

$$\begin{pmatrix} 1 & 5 & 0 & 1 & 0 \\ 0 & 5 & 0 & 1 & 0 \\ 0 & 2 & 1 & 4 & 0 \\ 0 & 2 & 0 & 4 & -1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

In[39]:= **MatrixForm**[**Inverse**[B]]

Out[39]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & -\frac{1}{4} \\ 0 & \frac{1}{2} & 1 & 0 & -\frac{9}{2} \\ 0 & -\frac{1}{4} & 0 & 0 & \frac{5}{4} \\ 0 & -\frac{1}{2} & 0 & -1 & \frac{9}{2} \end{pmatrix}$$

In[43]:= **(\*Dual Simplex Method\*)**

**MatrixForm**[

**C1 =** {{5, 1, 3, 1, 0, 0, 0, 0, 0, 4}, {-5, -1, -3, 0, 1, 0, 0, 0, 0, -2}, {2, 4, 7, 0, 0, 0, 1, 0, 0, 1},  
 0, 0, 0, 5}, {-2, -4, -7, 0, 0, 0, 1, 0, 0, -3}, {1, 1, 1, 0, 0, 0, 0, 1, 0, 1},  
 {-1, -1, -1, 0, 0, 0, 0, 0, 1, -1}, {8, 6, 11, 0, 0, 0, 0, 0, 0, 0}}

Out[43]//MatrixForm=

$$\begin{pmatrix} 5 & 1 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 4 \\ -5 & -1 & -3 & 0 & 1 & 0 & 0 & 0 & 0 & -2 \\ 2 & 4 & 7 & 0 & 0 & 1 & 0 & 0 & 0 & 5 \\ -2 & -4 & -7 & 0 & 0 & 0 & 1 & 0 & 0 & -3 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 8 & 6 & 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[44]:= **MatrixForm**[**C2 = Pivot**[C1, 4, 2]]

Out[44]//MatrixForm=

$$\begin{pmatrix} \frac{9}{2} & 0 & \frac{5}{4} & 1 & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{13}{4} \\ -\frac{9}{2} & 0 & -\frac{5}{4} & 0 & 1 & 0 & -\frac{1}{4} & 0 & 0 & -\frac{5}{4} \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 2 \\ \frac{1}{2} & 1 & \frac{7}{4} & 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & \frac{3}{4} \\ \frac{1}{2} & 0 & -\frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 1 & 0 & \frac{1}{4} \\ -\frac{1}{2} & 0 & \frac{3}{4} & 0 & 0 & 0 & -\frac{1}{4} & 0 & 1 & -\frac{1}{4} \\ 5 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & -\frac{9}{2} \end{pmatrix}$$

In[45]:= **MatrixForm[C3 = Pivot[C2, 2, 3]]**

Out[45]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 2 \\ \frac{18}{5} & 0 & 1 & 0 & -\frac{4}{5} & 0 & \frac{1}{5} & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 2 \\ -\frac{29}{5} & 1 & 0 & 0 & \frac{7}{5} & 0 & -\frac{3}{5} & 0 & 0 & -1 \\ \frac{16}{5} & 0 & 0 & 0 & -\frac{3}{5} & 0 & \frac{2}{5} & 1 & 0 & 1 \\ -\frac{16}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & -\frac{2}{5} & 0 & 1 & -1 \\ \frac{16}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & \frac{7}{5} & 0 & 0 & -5 \end{pmatrix}$$

In[46]:= **MatrixForm[C4 = Pivot[C3, 4, 1]]**

Out[46]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & \frac{18}{29} & 1 & 0 & \frac{2}{29} & 0 & -\frac{5}{29} & 0 & 0 & \frac{11}{29} \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 2 \\ 1 & -\frac{5}{29} & 0 & 0 & -\frac{7}{29} & 0 & \frac{3}{29} & 0 & 0 & \frac{5}{29} \\ 0 & \frac{16}{29} & 0 & 0 & \frac{5}{29} & 0 & \frac{2}{29} & 1 & 0 & \frac{13}{29} \\ 0 & -\frac{16}{29} & 0 & 0 & -\frac{5}{29} & 0 & -\frac{2}{29} & 0 & 1 & -\frac{13}{29} \\ 0 & \frac{16}{29} & 0 & 0 & \frac{34}{29} & 0 & \frac{31}{29} & 0 & 0 & -\frac{161}{29} \end{pmatrix}$$

In[47]:= **MatrixForm[C5 = Pivot[C4, 6, 2]]**

Out[47]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -\frac{1}{8} & 0 & -\frac{1}{4} & 0 & \frac{9}{8} & -\frac{1}{8} \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & -\frac{3}{16} & 0 & \frac{1}{8} & 0 & -\frac{5}{16} & \frac{5}{16} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & \frac{5}{16} & 0 & \frac{1}{8} & 0 & -\frac{29}{16} & \frac{13}{16} \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & -6 \end{pmatrix}$$

In[48]:= **MatrixForm[C6 = Pivot[C5, 2, 7]]**

Out[48]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & -4 & 0 & \frac{1}{2} & 0 & 1 & 0 & -\frac{9}{2} & \frac{1}{2} \\ 0 & 0 & 4 & 0 & -\frac{1}{2} & 1 & 0 & 0 & \frac{9}{2} & \frac{3}{2} \\ 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 & -\frac{5}{4} & \frac{3}{4} \\ 0 & 0 & 4 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{11}{2} & -\frac{13}{2} \end{pmatrix}$$