

```

In[1]:= Pivot[A_, k_, l_] := Module[{B = A},
  m = Dimensions[A][[1]]; n = Dimensions[A][[2]];
  For[i = 1, i ≤ m, i++,
    If[i == k,
      B[[i]] = A[[i]] / A[[k, l]],
      B[[i]] = -A[[i, l]] / A[[k, l]] * A[[k]] + A[[i]]
    ];
  B
]

```

(\*Section 2.3 Example 2\*)  
 (\*2-phase method\*)

```

In[2]:= MatrixForm[A0 = {{1, 2, 2, 1, 1, 0, 1, 0, 12},
  {1, 2, 1, 1, 2, 1, 0, 0, 18}, {3, 6, 2, 1, 3, 0, 0, 1, 24}, {0, 0, 0, 0, 0, 0, 1, 1, 0}}]

```

Out[2]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 2 & 1 & 1 & 0 & 1 & 0 & 12 \\ 1 & 2 & 1 & 1 & 2 & 1 & 0 & 0 & 18 \\ 3 & 6 & 2 & 1 & 3 & 0 & 0 & 1 & 24 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

```

In[3]:= A1 = A0; A1[[4]] = A0[[4]] - A0[[1]] - A0[[3]];
  MatrixForm[A1]

```

Out[4]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 2 & 1 & 1 & 0 & 1 & 0 & 12 \\ 1 & 2 & 1 & 1 & 2 & 1 & 0 & 0 & 18 \\ 3 & 6 & 2 & 1 & 3 & 0 & 0 & 1 & 24 \\ -4 & -8 & -4 & -2 & -4 & 0 & 0 & 0 & -36 \end{pmatrix}$$

```

In[5]:= MatrixForm[A2 = Pivot[A1, 3, 2]]

```

Out[5]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{4}{3} & \frac{2}{3} & 0 & 0 & 1 & -\frac{1}{3} & 4 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 1 & 0 & -\frac{1}{3} & 10 \\ \frac{1}{2} & 1 & \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 & 0 & \frac{1}{6} & 4 \\ 0 & 0 & -\frac{4}{3} & -\frac{2}{3} & 0 & 0 & 0 & \frac{4}{3} & -4 \end{pmatrix}$$

```

In[7]:= MatrixForm[A3 = Pivot[A2, 1, 3]]

```

Out[7]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & \frac{3}{4} & -\frac{1}{4} & 3 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 1 & -\frac{1}{4} & -\frac{1}{4} & 9 \\ \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{4} & \frac{1}{4} & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

```
In[8]:= B0 = Drop[A3, None, {7, 8}];
B0[[4]] = {-1, 2, 3, 1, 1, -2, 0};
MatrixForm[B0]
```

Out[10]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & 3 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 1 & 9 \\ \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & 3 \\ -1 & 2 & 3 & 1 & 1 & -2 & 0 \end{pmatrix}$$

```
In[11]:= B1 = B0; B1[[4]] = B0[[1]]*(-3) + B0[[2]]*2 + B0[[3]]*(-2) + B0[[4]];
MatrixForm[B1]
```

Out[12]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & 3 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 1 & 9 \\ \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & 3 \\ -2 & 0 & 0 & \frac{1}{2} & 2 & 0 & 3 \end{pmatrix}$$

```
In[13]:= MatrixForm[B2 = Pivot[B1, 3, 1]]
```

Out[13]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & 3 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 1 & 9 \\ 1 & 2 & 0 & 0 & 1 & 0 & 6 \\ 0 & 4 & 0 & \frac{1}{2} & 4 & 0 & 15 \end{pmatrix}$$

(\*Section 2.3 Example 2\*)

(\*Big M\*)

```
MatrixForm[C0 = {{1, 2, 2, 1, 1, 0, 1, 0, 12}, {1, 2, 1, 1, 2, 1, 0, 0, 18},
{3, 6, 2, 1, 3, 0, 0, 1, 24}, {-1, 2, 3, 1, 1, -2, M, M, 0}}]
```

Out[20]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 2 & 1 & 1 & 0 & 1 & 0 & 12 \\ 1 & 2 & 1 & 1 & 2 & 1 & 0 & 0 & 18 \\ 3 & 6 & 2 & 1 & 3 & 0 & 0 & 1 & 24 \\ -1 & 2 & 3 & 1 & 1 & -2 & M & M & 0 \end{pmatrix}$$

```
In[21]:= C1 = C0; Simplify[C1[[4]] = C0[[4]] + C0[[1]]*(-M) + C0[[2]]*2 + C0[[3]]*(-M)];
MatrixForm[C1]
```

Out[22]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 2 & 1 & 1 & 0 & 1 & 0 & 12 \\ 1 & 2 & 1 & 1 & 2 & 1 & 0 & 0 & 18 \\ 3 & 6 & 2 & 1 & 3 & 0 & 0 & 1 & 24 \\ 1 - 4M & 6 - 8M & 5 - 4M & 3 - 2M & 5 - 4M & 0 & 0 & 0 & 36 - 36M \end{pmatrix}$$

In[25]:= **MatrixForm[C2 = Simplify[Pivot[C1, 3, 2]]]**

Out[25]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{4}{3} & \frac{2}{3} & 0 & 0 & 1 & -\frac{1}{3} & 4 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 1 & 0 & -\frac{1}{3} & 10 \\ \frac{1}{2} & 1 & \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 & 0 & \frac{1}{6} & 4 \\ -2 & 0 & 3 - \frac{4M}{3} & 2 - \frac{2M}{3} & 2 & 0 & 0 & -1 + \frac{4M}{3} & -4(-3 + M) \end{pmatrix}$$

In[26]:= **MatrixForm[C3 = Simplify[Pivot[C2, 1, 3]]]**

Out[26]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & \frac{3}{4} & -\frac{1}{4} & 3 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 1 & -\frac{1}{4} & -\frac{1}{4} & 9 \\ \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{4} & \frac{1}{4} & 3 \\ -2 & 0 & 0 & \frac{1}{2} & 2 & 0 & -\frac{9}{4} + M & -\frac{1}{4} + M & 3 \end{pmatrix}$$

In[27]:= **MatrixForm[C4 = Simplify[Pivot[C3, 3, 1]]]**

Out[27]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & \frac{3}{4} & -\frac{1}{4} & 3 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 1 & -\frac{1}{4} & -\frac{1}{4} & 9 \\ 1 & 2 & 0 & 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 6 \\ 0 & 4 & 0 & \frac{1}{2} & 4 & 0 & -\frac{13}{4} + M & \frac{3}{4} + M & 15 \end{pmatrix}$$

(\*2.3: Exercise 21\*)

(\*Slack variables: x5,x6; Artificial variables: x7, x8\*)

(\*big-M method\*)

In[28]:= **D0 = {{0, 1, 7, 2, -1, 0, 1, 0, 3}, {1, 2, 1, 0, 0, 0, 0, 1, 9},**  
**{2, 3, 1, -4, 0, 1, 0, 0, 7}, {-3, 1, -2, -4, 0, 0, M, M, 0}};**

**MatrixForm[D0]**

Out[29]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 7 & 2 & -1 & 0 & 1 & 0 & 3 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 9 \\ 2 & 3 & 1 & -4 & 0 & 1 & 0 & 0 & 7 \\ -3 & 1 & -2 & -4 & 0 & 0 & M & M & 0 \end{pmatrix}$$

In[30]:= **D1 = D0;**

**D1[[4]] = D0[[1]]\*(-M) + D0[[2]]\*(-M) + D0[[4]];**

**MatrixForm[D1]**

Out[32]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 7 & 2 & -1 & 0 & 1 & 0 & 3 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 9 \\ 2 & 3 & 1 & -4 & 0 & 1 & 0 & 0 & 7 \\ -3 - M & 1 - 3M & -2 - 8M & -4 - 2M & M & 0 & 0 & 0 & -12M \end{pmatrix}$$

In[36]:= **MatrixForm[D2 = Simplify[Pivot[D1, 1, 3]]]**

Out[36]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{1}{7} & 1 & \frac{2}{7} & -\frac{1}{7} & 0 & \frac{1}{7} & 0 & \frac{3}{7} \\ 1 & \frac{13}{7} & 0 & -\frac{2}{7} & \frac{1}{7} & 0 & -\frac{1}{7} & 1 & \frac{60}{7} \\ 2 & \frac{20}{7} & 0 & -\frac{30}{7} & \frac{1}{7} & 1 & -\frac{1}{7} & 0 & \frac{46}{7} \\ -3 - M & \frac{1}{7} (9 - 13 M) & 0 & \frac{2}{7} (-12 + M) & \frac{1}{7} (-2 - M) & 0 & \frac{2}{7} (1 + 4 M) & 0 & \frac{6}{7} - \frac{60 M}{7} \end{pmatrix}$$

In[38]:= **MatrixForm[D3 = Simplify[Pivot[D2, 3, 2]]]**

Out[38]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{10} & 0 & 1 & \frac{1}{2} & -\frac{3}{20} & -\frac{1}{20} & \frac{3}{20} & 0 & \frac{1}{10} \\ -\frac{3}{10} & 0 & 0 & \frac{5}{2} & \frac{1}{20} & -\frac{13}{20} & -\frac{1}{20} & 1 & \frac{43}{10} \\ \frac{7}{10} & 1 & 0 & -\frac{3}{2} & \frac{1}{20} & \frac{7}{20} & -\frac{1}{20} & 0 & \frac{23}{10} \\ \frac{3}{10} (-13 + M) & 0 & 0 & \frac{1}{2} (-3 - 5 M) & \frac{1}{20} (-7 - M) & \frac{1}{20} (-9 + 13 M) & \frac{7}{20} (1 + 3 M) & 0 & \frac{1}{10} (-21 - 43 M) \end{pmatrix}$$

In[39]:= **MatrixForm[D4 = Simplify[Pivot[D3, 1, 4]]]**

Out[39]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{5} & 0 & 2 & 1 & -\frac{3}{10} & -\frac{1}{10} & \frac{3}{10} & 0 & \frac{1}{5} \\ \frac{1}{5} & 0 & -5 & 0 & \frac{4}{5} & -\frac{2}{5} & -\frac{4}{5} & 1 & \frac{19}{5} \\ \frac{2}{5} & 1 & 3 & 0 & -\frac{2}{5} & \frac{1}{5} & \frac{2}{5} & 0 & \frac{13}{5} \\ \frac{1}{5} (-21 - M) & 0 & 3 + 5 M & 0 & -\frac{4}{5} (1 + M) & \frac{1}{5} (-3 + 2 M) & \frac{1}{5} (4 + 9 M) & 0 & \frac{1}{5} (-9 - 19 M) \end{pmatrix}$$

In[40]:= **MatrixForm[D5 = Simplify[Pivot[D4, 2, 5]]]**

Out[40]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{8} & 0 & \frac{1}{8} & 1 & 0 & -\frac{1}{4} & 0 & \frac{3}{8} & \frac{13}{8} \\ \frac{1}{4} & 0 & -\frac{25}{4} & 0 & 1 & -\frac{1}{2} & -1 & \frac{5}{4} & \frac{19}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{9}{2} \\ -4 & 0 & -2 & 0 & 0 & -1 & M & 1 + M & 2 \end{pmatrix}$$