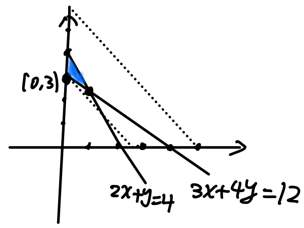
2.3 : 15 **Minimize** $3x + 2.5y$

subject to

$$3x + 4y \geq 12$$
$$2x + y \leq 4$$
$$x, y \geq 0.$$



$\Leftrightarrow$ Maximize $-3x - 2.5y$

Subject to

$$-3x - 4y \leq -12 \quad \rightsquigarrow \quad -3x - 4y + u = -12$$
$$2x + y \leq 4 \qquad\qquad 2x + y + v = 4$$
$$x, y \geq 0. \qquad\qquad x, y, u, v \geq 0.$$

$$A = \begin{pmatrix} -3 & -4 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} -12 \\ 4 \end{pmatrix}.$$

basic solution $(\underbrace{0, 0}_{\text{non basic}}, \underbrace{-12, 4}_{\text{basic}}) \leftarrow$

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$B^{-1} \cdot b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -12 \\ 4 \end{pmatrix} = \begin{pmatrix} -12 \\ 4 \end{pmatrix}$$

Two-Phase Method.

Phase 1: Find an initial extremal point = basic feasible solution

Phase 2: Apply Simplex Method to extremal point from Phase 1.

$$3x + 4y \geq 12 \qquad\qquad 3x + 4y - u = 12 \qquad\qquad 3x + 4y - u + w = 12$$
$$2x + y \leq 4 \quad \Rightarrow \quad 2x + y + v = 4 \quad \rightsquigarrow \quad 2x + y + v = 4$$
$$x, y \geq 0.$$

$$\begin{array}{ccccc} x & y & u & v & w \end{array}$$
$$\boxed{\begin{pmatrix} 3 & 4 & -1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 \end{pmatrix}}$$

Phase 1: Minimize $z = w \geq 0$

$$\left.\begin{array}{l} \text{subject to } 3x+4y-u+w=12 \\ \quad 2x+y+v = 4 \\ \quad x, y, u, v, w \geq 0 \end{array}\right\} LP_1$$

basic feasible $(x, y, u, v) \Longleftrightarrow (x, y, u, v, 0)$ Optimal solution to LP_1

	x	y ↓	u	v	w	
← w	3	4	−1	0	1	12
v	2	1	0	1	0	4
	−3	−4	1	0	0	−12
→	~~0~~	~~0~~	~~0~~	~~0~~	~~1~~	~~0~~

Maximize $z' = -w$

$\Updownarrow$

$z' + w = 0$

$z' = -w$

$= 3x + 4y - u - 12$

	x	y	u	v	w	
y	$\frac{3}{4}$	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	3
v	$\frac{5}{4}$	0	$\frac{1}{4}$	1	$-\frac{1}{4}$	1
	0	0	0	0	1	0

Optimal solution

$\leadsto (x, y, u, v, w)$

$\quad = (0, 3, 0, 1, 0)$.

$\Downarrow$

$(x, y, u, v) = (0, 3, 0, 1)$

$\Downarrow$

$(x, y) = (0, 3)$

Minimize $3x + 2.5y \longrightarrow$ Maximize $-3x - 2.5y$

	x	y	u	v	~~w~~	
y	$\frac{3}{4}$	1	$-\frac{1}{4}$	0	~~$\frac{1}{4}$~~	3
v	$\frac{5}{4}$	0	$\frac{1}{4}$	1	~~$-\frac{1}{4}$~~	1
	~~0~~	~~0~~	~~0~~	~~0~~	~~1~~	0
$\rightarrow$	3	$\frac{5}{2}$	0	0		0
(y row) $\times(-\frac{5}{2})$	$-\frac{15}{8}$	$-\frac{5}{2}$	$\frac{5}{8}$	0		$-\frac{15}{2}$

$\frac{3}{4}\times(-\frac{5}{2})$ $\quad -\frac{1}{4}\times(-\frac{5}{2})$ $\quad 3\times(-\frac{5}{2})$

$\rightarrow$

	x	y	u	v	
y	$\frac{3}{4}$	1	$-\frac{1}{4}$	0	3
v	$\frac{5}{4}$	0	$\frac{1}{4}$	1	1
	$\frac{9}{8}$	0	$\frac{5}{8}$	0	$-\frac{15}{2}$

$\Rightarrow$ Optimal solution

$(x, y, u, v) = (0, 3, 0, 1)$

Minimum $= -(-\frac{15}{2}) = \frac{15}{2}$