Maximize $z = 2x_1 + 3x_2 - x_3 \implies z - 2x_1 - 3x_2 + x_3 = 0.$

Subject to
$$x_1 + 2x_2 - x_3 \leq 6$$
$$x_1 - 3x_2 - 3x_3 \leq 10$$
$$x_j \geq 0, \; j = 1, 2, 3.$$

Sol:
$$x_1 + 2x_2 - x_3 + x_4 = 6$$
$$x_1 - 3x_2 - 3x_3 + x_5 = 10$$

$$x_i \geq 0, \; i = 1, \cdots, 5.$$

$\downarrow$

	x_1	x_2	x_3	x_4	x_5	
$\Leftarrow$ x_4	1	②	-1	1	0	6
x_5	1	-3	-3	0	1	10
	-2	-3	1	0	0	0

	x_1	x_2	x_3	x_4	x_5	
x_2	$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	3
x_5	$\frac{5}{2}$	0	$-\frac{9}{2}$	$\frac{3}{2}$	1	19
	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{3}{2}$	0	9

basic solution $(0, 0, 0, 6, 10)$

$\Uparrow (0, 3, 0, 0, 19).$

If x_1 enters, then

θ-ratio: $x_2 : \dfrac{3}{\frac{1}{2}} = 6$

$x_5 : \dfrac{19}{\frac{5}{2}} = \dfrac{38}{5} > 6.$

If x_3 enters, then.

$x_1, \; x_4$ non-basic.
$\underset{0}{\overset{\shortparallel}{}} \quad \underset{0}{\overset{\shortparallel}{}}$

$\leadsto$

$$\begin{cases} \frac{1}{2}x_1 + x_2 - \frac{1}{2}x_3 + \frac{1}{2}x_4 = 3 \\ \frac{5}{2}x_1 - \frac{9}{2}x_3 + \frac{3}{2}x_4 + x_5 = 19. \end{cases}$$

$$z - \frac{1}{2}x_1 - \frac{1}{2}x_3 + \frac{3}{2}x_4 = 9$$
$\Updownarrow$
$$z = \frac{1}{2}x_1 + \frac{1}{2}x_3 - \frac{3}{2}x_4 + 9.$$

$$\begin{cases} x_2 - \frac{1}{2}x_3 = 3 \\ -\frac{9}{2}x_3 + x_5 = 19 \\ x_i \geq 0. \end{cases}$$

$\implies x_3$ unbounded $\implies$ no optimal solution.

Maximize $z = 5x_1 + 3x_2$

subject to $\boxed{x_1 - x_2 \leq 2}$

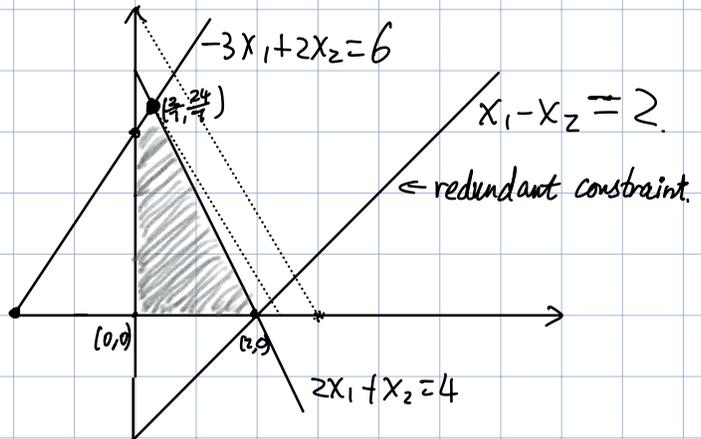$2x_1 + x_2 \leq 4$

$-3x_1 + 2x_2 \leq 6$

$x_1, x_2 \geq 0$

$\rightsquigarrow$

$x_1 - x_2 + x_3 = 2$

$2x_1 + x_2 + x_4 = 4$

$-3x_1 + 2x_2 + x_5 = 6.$

$x_i \geq 0, \; i = 1, \cdots, 5.$

	x_1	x_2	x_3	x_4	x_5	
x_3	①	-1	1	0	0	2
x_4	2	1	0	1	0	4
x_5	-3	2	0	0	1	6
	-5	-3	0	0	0	

$\frac{3}{11} \quad \frac{5}{17}$

$m \times n$

$-3x_1 + 2x_2 = 6$

$(\frac{3}{7}, \frac{24}{7})$

$x_1 - x_2 = 2.$

$\Leftarrow$ redundant constraint.

$(0,0)$ $(2,0)$

$2x_1 + x_2 = 4$

Tableau 2.13 - 2.16:

	$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$		
basic variables	basic feasible solution	z	extremal point
$x_3 \quad x_4 \quad x_5$	$(0, 0, 2, 4, 6)$	0	$(0, 0)$
$x_1 \quad x_4 \quad x_5$	$(2, 0, 0, 0, 12)$	10	$(2, 0)$
$x_1 \quad x_2 \quad x_5$	$(2, 0, 0, 0, 12)$	10	$(2, 0)$
$x_1 \quad x_2 \quad x_3$	$(\frac{2}{7}, \frac{24}{7}, \frac{36}{7}, 0, 0)$	$\frac{82}{7}$	$(\frac{2}{7}, \frac{24}{7})$

Tableau 2.13a - 2.15a:

	$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$		
basic variables	basic feasible solution	z	extremal point
$x_3 \quad x_4 \quad x_5$	$(0, 0, 2, 4, 6)$	0	$(0, 0)$
$x_3 \quad x_1 \quad x_5$	$(2, 0, 0, 0, 12)$	10	$(2, 0)$
$x_3 \quad x_1 \quad x_2$	$(\frac{2}{7}, \frac{24}{7}, \frac{36}{7}, 0, 0)$	$\frac{82}{7}$	$(\frac{2}{7}, \frac{24}{7}).$
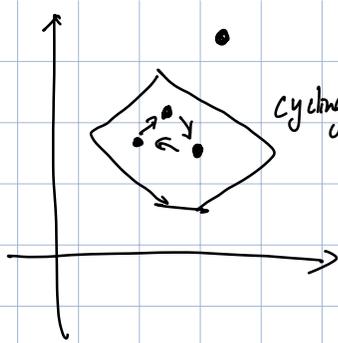
cycling $\longrightarrow$ Bland rule $\leadsto$ choose columns rows with the smallest subscript.