$$A=\begin{pmatrix}3&1&-2\\-1&0&5\\-1&-1&4\end{pmatrix}\qquad f(\lambda)=|A-\lambda I|=-(\lambda-2)^2(\lambda-3).$$

1. Find its Jordan canonical form.

$\lambda=2$: $$A-\lambda I=\begin{pmatrix}1&1&-2\\-1&-2&5\\-1&-1&2\end{pmatrix}\to\begin{pmatrix}1&1&-2\\0&-1&3\\0&0&0\end{pmatrix}\to\begin{pmatrix}1&0&1\\0&1&-3\\0&0&0\end{pmatrix}$$

$$\Leftrightarrow\begin{cases}x_1+x_3=0\\x_2-3x_3=0\end{cases}\Rightarrow \underset{\overset{\shortparallel}{E_2}}{N(A-2I)}=\operatorname{Span}\left\{\begin{pmatrix}-1\\3\\1\end{pmatrix}\right\},\ \dim E_2=1<\underset{\overset{\shortparallel}{2}}{\operatorname{mult}(2)}$$

$\Rightarrow$ Dot diagram for $\lambda=2$: $\begin{array}{c}\bullet\\\bullet\end{array}$, Dot diagram for $\lambda=3$ is $\bullet$

$\Rightarrow$ Jordan canonical form of A is. $\begin{pmatrix}\boxed{\begin{matrix}2&1\\0&2\end{matrix}}&\\&\boxed{3}\end{pmatrix}$

2. Find S s.t. $S^{-1}AS$ is its Jordan canonical form.

$\lambda=2$: $\begin{array}{c}\bullet\, v_1\\\uparrow\\\bullet\, v_2\end{array}$ choose $v_1=\begin{pmatrix}-1\\3\\1\end{pmatrix}$ solve $(A-2I)v_2=v_1$

$$\left(\begin{array}{ccc|c}1&1&-2&-1\\-1&-2&5&3\\-1&-1&2&1\end{array}\right)\to\left(\begin{array}{ccc|c}1&1&-2&-1\\0&-1&3&2\\0&0&0&0\end{array}\right)\to\left(\begin{array}{ccc|c}1&0&1&1\\0&1&-3&-2\\0&0&0&0\end{array}\right)$$

$\Rightarrow$ choose $v_2=\begin{pmatrix}1\\-2\\0\end{pmatrix}$

$$\lambda=3:\ A-3I=\begin{pmatrix}0&1&-2\\-1&-3&5\\-1&-1&1\end{pmatrix}\to\begin{pmatrix}1&1&-1\\0&1&-2\\0&-2&4\end{pmatrix}\to\begin{pmatrix}1&0&1\\0&1&-2\\0&0&0\end{pmatrix}\Rightarrow v_3=\begin{pmatrix}-1\\2\\1\end{pmatrix}$$

$$\Rightarrow S=(v_1\ v_2\ v_3)=\begin{pmatrix}-1&1&-1\\3&-2&2\\1&0&1\end{pmatrix}\text{ satisfies } S^{-1}AS=J=\begin{pmatrix}\boxed{\begin{matrix}2&1\\0&2\end{matrix}}&\begin{matrix}0\\0\end{matrix}\\\begin{matrix}0&0\end{matrix}&\boxed{3}\end{pmatrix}$$