1. Define vector addition and scalar multiplication for

$$V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \right\}$$

to make it a vector space.

Sol: $$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{pmatrix}$$ vector addition

$$\lambda \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{pmatrix}$$ scalar multiplication.

<u>Note that</u>: $V \cong \mathbb{R}^4$.

2. Let $S_1 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc \neq 0 \right\}$

$S_2 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = 0 \right\}$

Are these subsets subspaces of V ?

Sol: For S_1, No: $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \notin S_1$

For S_2, No: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \in S_2$, $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \in S_2$

but $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \notin S_2$.

Neither is a subspace of V.