

MATH 350

FALL 2023

MIDTERM II

NAME:

ID:

THERE ARE FOUR (4) PROBLEMS. THEY HAVE THE INDICATED VALUE.

SHOW YOUR WORK

NO CALCULATORS NO CELLS ETC.

ON YOUR DESK: ONLY test, pen, pencil, eraser.

1		25pts
2		30pts
3		25pts
4		20pts
Total		100pts

!!! WRITE YOUR NAME, STUDENT ID. BELOW !!!

NAME :

ID :

1(25pts) A is a 3×5 matrix whose reduced row ethelon form is equal to:

$$\begin{pmatrix} 1 & 0 & -2 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

- (1) Find a basis for the null space $N(A)$.
- (2) Assume that $A = (v_1 \ v_2 \ v_3 \ v_4 \ v_5)$ where v_i denotes the i -th column.

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \quad v_4 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Recover the matrix A . (Be careful of the subscripts of column indices)

2(30pts) Let $T : P_2(\mathbf{R}) \rightarrow P_2(\mathbf{R})$ be the linear transformation given by

$$T(f) = x^2 f''(x) + f'(x) + 2f(x).$$

Let $\beta = \{1, x, x^2\}$ be the standard basis.

- (1) Calculate the matrix representation $A = [T]_\beta$ w.r.t. the standard basis β .
- (2) Determine whether T is diagonalizable or not. If yes, then find Q such that $Q^{-1}AQ$ is diagonal.

3(25pts) Let V be the subspace of $P_3(\mathbf{R})$ spanned by

$$1 + x + x^2, \quad 2x + x^3, \quad 2 + 2x^2 - x^3, \quad 1 + x^3$$

- (1) Find a basis for V .
- (2) Is $f(x) = 2 - x^2 + x^3$ in the subspace V ? Write down reason and calculations.

4(20pts) Assume that V is a vector space with a basis $\beta = \{v_1, v_2, v_3, v_4\}$. Let $T : V \rightarrow V$ be a linear transformation that satisfies:

$$Tv_1 = v_2, \quad Tv_2 = v_3, \quad Tv_3 = v_4, \quad Tv_4 = 2v_2 + v_3.$$

- (1) Calculate the characteristic polynomial of T .
- (2) The linear transformation T satisfies the identity:

$$T^4 - T^2 = a \cdot T + b \cdot \text{Id}_V$$

where Id_V is the identity transformation of V . Find the numbers a and b .

Continuation of work:

Scrap paper