MATH 350 FALL 2023 MIDTERM I

NAME: **ID:**

THERE ARE FOUR (4) PROBLEMS. THEY HAVE THE INDICATED VALUE.

SHOW YOUR WORK

NO CALCULATORS NO CELLS ETC.

ON YOUR DESK: ONLY test, pen, pencil, eraser.

1		20pts
2		30pts
3		20pts
4		30pts
Total		100pts

!!! WRITE YOUR NAME, STUDENT ID. BELOW !!!

NAME : **ID :**

1(20pts) Let $S = \{v_1, v_2, v_3, v_4\}$ be a subset of $\mathbf{R}^3$ with

$$v_1 = (1, 0, 1), v_2 = (0, 1, 0), v_3 = (1, -1, 1), v_4 = (1, 0, -1).$$

(1) Is S is a basis for $\mathbf{R}^3$? Why?
(2) Find a subset β of S such that β is a basis for $\mathbf{R}^3$.

2(30pts) Consider the linear transformation:

$$T : P_2(\mathbf{R}) \to P_2(\mathbf{R}), \quad T(f) = xf'(x) + f(x).$$

Let $\beta = \{1, x, x^2\}$, $\gamma = \{1, 1+x, x+x^2\}$.

(1) Find the matrix representation $[T]_\gamma^\beta$
(2) Find the matrix representation $[T]_\beta^\gamma$.
(3) Let $f(x) = 1 + x^2$. Calculate $[T(f(x))]_\gamma$.

3(20 pts) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation defined as:

$$T(x, y, z) = (x + y + z, x + 2y + 3z, x + 5y + 9z).$$

Determine whether T is one-to-one or onto.

4(30pts) Let $T : V \to W$ be a linear transformation between finite dimensional vector spaces. Let $\beta = \{v_1, \ldots, v_n\}$ be a basis for V.

(1) Assume that there a linear transformation $S : W \to V$ such that $T \circ S = \mathrm{Id}_W$. Must T be one-to-one or onto? Prove your claim.

(2) Assume that $\{T(v_1), \ldots, T(v_n)\}$ is a linearly independent subset of W. Must T be one-to-one or onto? Prove your claim.

Continuation of works:

Scrap paper