MATH 350 FALL 2023 FINAL EXAM

NAME: **ID:**

THERE ARE NINE (9) PROBLEMS. THEY HAVE THE INDICATED VALUE.

SHOW YOUR WORK

NO CALCULATORS NO CELLS ETC.

ON YOUR DESK: ONLY test, pen, pencil, eraser.

1		15pts
2		20pts
3		25pts
4		25pts
5		25pts
6		20pts
7		25pts
8		25pts
9		20pts
Total		200pts

!!! WRITE YOUR NAME, STUDENT ID. BELOW !!!

NAME : **ID :**

1(15pts) For each of the following subsets of $\mathbf{R}^3$, determine whether it is a vector subspace of $\mathbf{R}^3$. Explain your reason.

(1) $\{(a, b, c); a + 99b = 101c\}$.
(2) $\{(a, b, c); a^2 - b^2 = 0\}$.
(3) $\{(a, b, c); a^2 + b^2 = 0\}$.

2(25pts) Consider the following subset S of $P_3(\mathbf{R})$.

$$S = \{1 - x, x + x^2, x^2 + x^3, 1 + x^3\}$$

(1) Is S linearly dependent or linearly independent?
(2) Is $x + x^2 + 2x^3$ contained in Span(S)? Explain your reason.

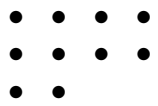
3(20pts) Let r_1, r_2, r_3 be row vectors in $\mathbf{R}^3$. Find the value of k that satisfies the following equality:

$$\det \begin{pmatrix} 3r_1 + r_2 \\ r_1 + r_3 \\ r_1 + 2r_2 \end{pmatrix} = k \cdot \det \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix}$$

4(25pts) Let A be a real square matrix. Assume that v_1 and v_2 are two eigenvectors associated with two different eigenvalues $\lambda_1 \neq \lambda_2$. Prove that v_1, v_2 are linearly independent. What else can you say if moreover A is symmetric?

5(25pts) (1) Use the Gram-Schmidt process to $\{1, x, x^2\}$ to find an orthonomal basis β of $P_2(\mathbf{R})$ with respect to the inner product $\langle f, g\rangle = \int_{-2}^{2} f(x)g(x)dx$.
(2) Find the Fourier coefficients of $h(x) = x^2$ with respect to β.

6(25pts) Let A be a square matrix with characteristic polynomial equal to $(t-2)^{10}$. Assume that the dot diagram of A is the following:



(1) Write down the Jordan canonical form of A.
(2) Calculate $\dim R(A-2I)$ and $\dim R((A-2I)^2)$.

7(25pts) Consider the linear transformation:

$$T : P_2(\mathbf{R}) \to P_2(\mathbf{R}), \quad T(f(x)) = f''(x) + f'(x) + f(0).$$

(1) Find all eigenvalues of T and the corresponding dot diagrams.
(2) Find a basis γ of $P_2(\mathbf{R})$ such that $[T]_\gamma$ is a Jordan canonical form.

8(20pts) Find the linear function $f(t) = c_0 + c_1 t$ that has the best fit to the data:

$$\{(0,0),(1,2),(2,1),(3,-1),(4,0)\} = \{(t_i, y_i); 1 \leq i \leq 5\}.$$

with respect to the error: $\mathbf{E} = \sum_{i=1}^{5} (c_0 + c_1 t_i - y_i)^2$.

9(20pts) Let $S : V \to W$ and $T : W \to V$ be linear transformations between two vector spaces V and W. Let $T \circ S : V \to V$ be the composition.

(1) Prove that if S is surjective (i.e onto), then $\text{Rank}(T \circ S) = \text{Rank}(T)$.
(2) If S is injective (i.e. one-to-one), is $\text{Rank}(T \circ S) = \text{Rank}(T)$ always true? Prove this statement or find a counterexample.

Extra page

Extra page