Quiz 1:

1. Is $\mathbb{R}^2$ with the following operations a vector space?

$$(a_1, b_1) + (a_2, b_2) = (a_1 + b_2, b_1 + a_2)$$

$$c \cdot (a, b) = (ca, cb)$$

Sol: No. Because the vector addition defined above is not commutative:

$(a_1, b_1) + (a_2, b_2) = (a_1 + b_2,\ b_1 + a_2)$
$(a_2, b_2) + (a_1, b_1) = (a_2 + b_1,\ b_2 + a_1)$
are not equal in general.

for example, $(1,0) + (0,1) = (2,0)$
$\qquad\qquad\quad\ (0,1) + (1,0) = (0,2)$ #

2. Is $S = \{ f \in P(\mathbb{R}) : f(2) = 0 \}$ a subspace of $P(\mathbb{R})$?

Sol: Yes. Verify: $f, g \in S \Rightarrow f(2) = 0 = g(2)$

- S is closed under addition:

  $(f+g)(2) = f(2) + g(2) = 0. \Rightarrow f + g \in S.$

- S is closed under scalar multiplication: $c \in \mathbb{R}$,

  $(c \cdot f)(2) = c \cdot f(2) = c \cdot 0 = 0 \quad \Rightarrow c \cdot f \in S.$