MATH 350 SPRING 2023 MIDTERM I

NAME: **ID:**

THERE ARE FOUR (4) PROBLEMS. THEY HAVE THE INDICATED VALUE.

SHOW YOUR WORK

NO CALCULATORS NO CELLS ETC.

ON YOUR DESK: ONLY test, pen, pencil, eraser.

1		20pts
2		20pts
3		20pts
4		40pts
Total		100pts

!!! WRITE YOUR NAME, STUDENT ID. BELOW !!!

NAME : **ID :**

1(20pts) **(i)** Prove that $S = \{x^2 + 1, x^2 + x\} \subset P_2(\mathbf{R})$ is linearly independent.
(ii) Extend S to a basis β of $P_2(\mathbf{R})$. Explain why your extension is a basis.

2(20pts) Consider the linear transformation:

$$T : P_2(\mathbf{R}) \to M_{2\times 2}(\mathbf{R}), \quad T(f) = \begin{pmatrix} f(0) & f'(0) \\ f(1) & f'(1) \end{pmatrix}.$$

(1) Find the matrix representation of T with respect to the bases

$$\beta = \{1, x, x^2\}, \quad \gamma = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}.$$

(2) Let $f(x) = 3 - 2x + x^2$. Calculate $[f(x)]_\beta$ and $[T(f(x))]_\gamma$.

3(20 pts) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation defined as:

$$T(x, y, z) = (y - 2z, x - 2z, x - y).$$

Calculate nulllity(T) and rank(T).

4(40pts) Let $T : V \to W$ be a linear transformation between finite dimensional vector spaces. Assume that T is one-to-one but **not** onto.

(1) (20pts) Prove the inequality $\dim V < \dim W$.
(2) (10pts) Prove that there exists a linear transformation $U : W \to V$ such that $UT = \mathrm{Id}_V$ (Hint: define U by its values on basis vectors).
(3) (10pts) Is there a linear transformation $U : W \to V$ such that $TU = \mathrm{Id}_W$? Explain your reasons.

Continuation of works:

Scrap paper