HW1 Solution:

1.2: 9: Prove Cor. 1 and 2 of Thm 1.1 and Thm 1.2(c)

Proof of Cor. 1: Assume 0 and $0'$ satisfy (VS 3).

Then $\forall x \in V$, $x+0 = x+0'$.

By the Cancellation Law for Vector Addition (Thm 1.1), we get $0=0'$. ■

Proof of Cor. 2: Pick any $x \in V$. Assume y and y' both satisfy (VS4)

Then $x+y = 0 = x+y'$. By the Cancellation Law for Vector Addition

we get $y=y'$. So the negative of x is unique. ■

Pf of Thm 1.2(c) $\forall\, a \in F$, $\underset{\displaystyle a0+0}{\underset{\shortparallel}{a0}} = a\cdot(0+0) = a0 + a\cdot 0$

By the Cancellation Law, we get $a0=0$. ■

1.2.12: Prove that the set of even functions defined on the real line is a vector space.

Pf: This is a straight verification.

In fact, $W = \{\text{even functions}\}$ is a subspace of $\underset{\displaystyle \{\text{functions from } \mathbb{R} \text{ to } \mathbb{R}\}}{\underset{\shortparallel}{\mathcal{F}(\mathbb{R},\mathbb{R})}}$

- f, g even $\Rightarrow$ $f+g$ even
- f even, $a \in \mathbb{R} \Rightarrow a \cdot f$ is even
- 0 function is even.

1.2.17: $V=\{(a_1,a_2): a_1,a_2 \in F\}$.

$(a_1,a_2)+(b_1,b_2)=(a_1+b_1, a_2+b_2)$, $c\cdot(a_1,a_2)=(a_1,0)$

Is V a vector space over F?

Sol: (VS 1)–(VS 4) are satisfied.

Check (VS 5): $1\cdot(a_1,a_2)=(a_1,0)\neq(a_1,a_2)$ if $a_2\neq 0$.

So V violates (VS 5) and is NOT a vector space with this scalar mult.

Note that V satisfies (VS 6), (VS 7) but

violates (VS 8): $(c+d)(a_1,a_2)=(a_1,0)$ (if $a_1\neq 0$)

$\neq$

$c\cdot(a_1,a_2)+d(a_1,a_2)=(a_1,0)+(a_1,0)=(2a_1,0)$.

1.3.8(a) $W_1=\{(a_1,a_2,a_3)\in\mathbb{R}^3: a_1=3a_2 \text{ and } a_3=-a_2\}$.

W_1 is a subspace. This can be verified directly by showing that W_1 is closed under vector addition and scalar multiplication.

OR Note that

$W_1=\{(3a_2,a_2,-a_2): a_2\in\mathbb{R}\}$ = line passing through O and parallel to the vector $(3,1,-1)$.

$(3a_2,a_2,-a_2) = a_2\cdot(3,1,-1)$

1.3.8(e): $W_5 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 + 2a_2 - 3a_3 = 1\}$

W_5 is NOT a subspace. It is not closed under vector addition or the scalar multiplication and does not contain 0 vector.

Geometrically, W_5 is a plane that does not pass throug $0 = (0,0,0)$