Quiz: Solve the initial value Problem:

$$\begin{cases} y' = y + t + e^t \\ y(0) = 1 \end{cases}$$

Method 1: The general solution is $y = y_h + y_{p_1} + y_{p_2}$ ②

- y_h is the general solution to $y' = y \Rightarrow y_h = C \cdot e^t$ ②
- y_{p_1} is a particular solution to $y' = y + t$

  guess $y_{p_1} = at + b \Rightarrow y_{p_1}' = a \Rightarrow a = at + b + t = (a+1)t + b$

  $\Rightarrow \begin{cases} a+1 = 0 \\ b = a \end{cases} \Rightarrow a = -1 = b. \Rightarrow y_{p_1} = -t - 1$ ②

- y_{p_2} is a particular solution to $y' = y + e^t$

  guess $y_{p_2} = a \cdot t \cdot e^t$ ($a e^t$ does not work)

  $\Rightarrow y_{p_2}' = a e^t + a t \cdot e^t = a \cdot t \cdot e^t + e^t \Rightarrow a = 1$

  $\Rightarrow y_{p_2} = t \cdot e^t$ ②

So $y = y_h + y_{p_1} + y_{p_2} = C \cdot e^t - t - 1 + t \cdot e^t$

use initial value: $y(0) = C - 1 = 1 \Rightarrow C = 2.$ ②

So the unique solution is $y(t) = 2 \cdot e^t - t - 1 + te^t = (t+2)e^t - t - 1.$

Method 2 (Integrating factor): $y' - y = t + e^t$

$\mu(t) = e^{\int g(t)dt} = e^{\int (-1)dt} = e^{-t}$ ②

$\Rightarrow (e^{-t}y)' = (t+e^t)e^{-t} = t\cdot e^{-t} + 1$ ②

$\Rightarrow e^{-t}y = \int (t\cdot e^{-t} + 1)dt = \int t\cdot e^{-t}dt + t$ (integration by parts)

$= t - \int t\cdot d(e^{-t}) = t - (t\cdot e^{-t} - \int e^{-t}dt)$

$= t - t\cdot e^{-t} + \int e^{-t}dt = t - t\cdot e^{-t} - e^{-t} + C$ ②

$\Rightarrow y(t) = C\cdot e^t + t\cdot e^t - t - 1$ general solution ②

$y(0) = C - 1 = 1 \Rightarrow C = 2$ ②

$\Rightarrow y(t) = 2\cdot e^t + t\cdot e^t - t - 1 = (t+2)e^t - t - 1$