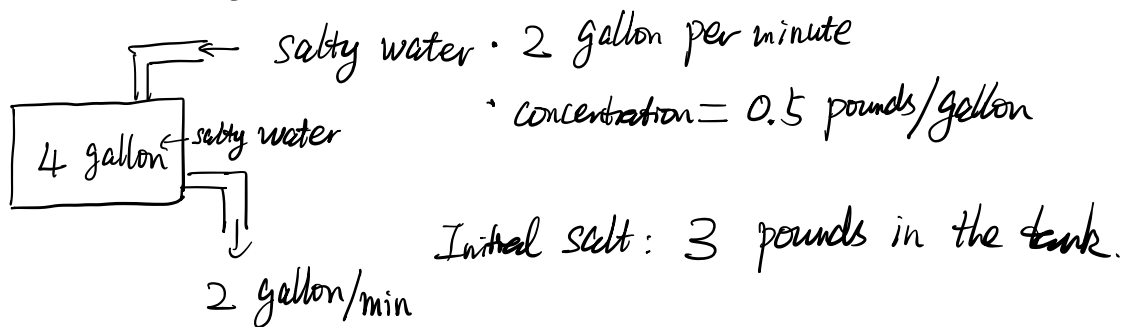


Quiz: Mixing Problem



- Question:
1. Find the amount of salt at time t in the tank.
 2. What is the amount of salt as $t \rightarrow +\infty$?
in the tank

Sol: y = amount of salt

$$1. \begin{cases} \frac{dy}{dt} = 0.5 \times 2 - \frac{y}{4} \times 2 = 1 - \frac{y}{2} = \frac{1}{2}(2-y) \\ y(0) = 3 \end{cases} \quad (2)$$

- Separate the variables:

$$\frac{dy}{2-y} = \frac{1}{2} dt \Rightarrow \int \frac{dy}{2-y} = \int \frac{1}{2} dt = \frac{1}{2} t + C_1 \quad (2)$$

-ln|2-y|

- use initial value: $-\ln|2-3| = \frac{1}{2} \times 0 + C_1 \Rightarrow C_1 = 0 \quad (2)$

$$\Rightarrow -\ln|2-y| = \frac{1}{2}t \Rightarrow |2-y| = e^{-\frac{1}{2}t}$$

$$y(0) = 3 > 2_{\text{equilibrium}} \Rightarrow y(t) > 2 \text{ for any } t$$

$$\Rightarrow y-2 = e^{-\frac{1}{2}t} \Rightarrow y(t) = 2 + e^{-\frac{1}{2}t} \quad (2)$$

$$\text{Verify: } \frac{dy}{dt} = -\frac{1}{2}e^{-\frac{1}{2}t} = 1 - \frac{1}{2}(2 + e^{-\frac{1}{2}t}), \quad y(0) = 3.$$

$$2. \quad \lim_{t \rightarrow \infty} y(t) = 2 + \lim_{t \rightarrow \infty} e^{-\frac{1}{2}t} = 2 + 0 = 2 \text{ pounds} \quad (2)$$