Matlab HW2

Assume that in a remote village with 1526 residents. A couple (2 persons) contracted a flu. The epidemic spread rapidly, as shown in the following table:

time	0	1	2	3	4	5	6	7	8	9	10	11	12	13
infected	2	7	22	71	194	384	492	446	336	231	152	97	62	39

Assume that the parameter $\beta = 0.56$ in the SIR model for the data in the above table.

1. Use technology to determine an appropriate value of α that matches the data in the table (hint: use Section 2.7, Problem 4(a) to help you).

   Solultion: By [Section 2.7, Problem 4(a)], the maximal of $I(S)$ is given by $\max I = f(\rho) = 1 - \rho + \rho \log \rho$ where $\rho = \frac{\beta}{\alpha}$. By using the above data, we get that $\max I = 492/1526$. So we would like to solve ρ from the identity:

   $$1 - \rho + \rho \log \rho = \frac{492}{1526}.$$

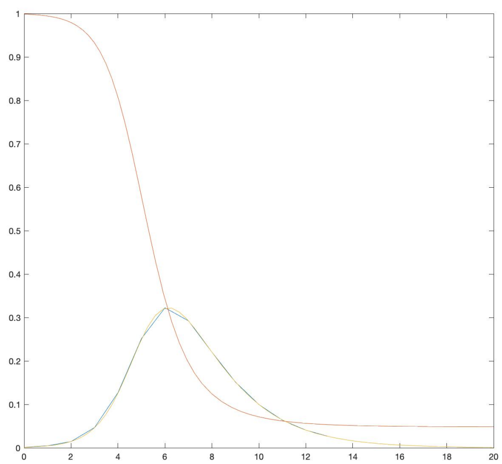
   This can be solved by using the MATLAB code:

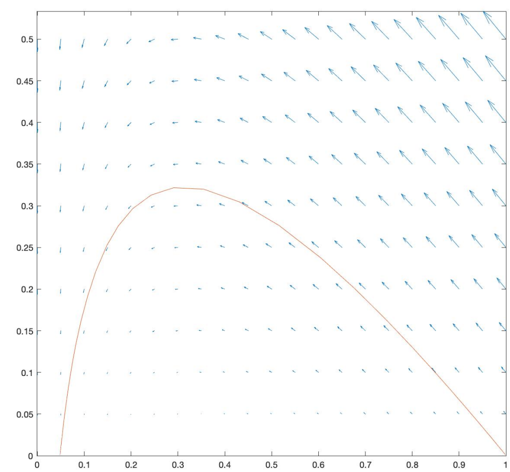
   ```
   fzero(@(x) 1-x+x*log(x)-492/1526, 0.5)
   ```

   and get the numerical answer $\rho = 0.3139$. So we get:

   $$\alpha = \frac{\beta}{\rho} = \frac{0.56}{0.3139} = 1.7840 \approx 1.78$$

2. Draw the solution curves (both the solution curve on the phase plane and the graphs of $I(t), S(t)$). Determine numerically the total number of residents who caught the flu during the epidemic.





Solution: We are considering the system:

$$\begin{cases} \frac{d}{dt}S(t) = -\alpha IS \\ \frac{d}{dt}I(t) = \alpha I - \beta I. \end{cases}$$

Use the attached MATLAB code with initial condition $(S(0), I(0)) = (\frac{1524}{1526}, \frac{2}{1526})$ to get numerical solutions of $I(t)$, $S(t)$. We also get the numerical value:

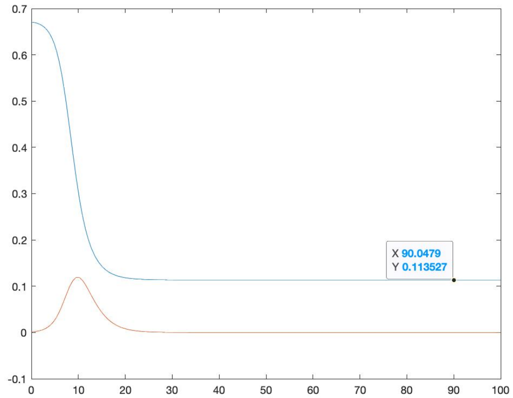
$$\lim_{t\to+\infty} S(t) \approx 0.048.$$

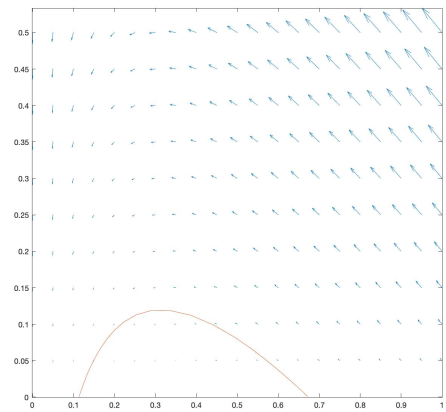
Or we can solve $I(S) = 0$ numerically by using the code:

```
fzero(@(x) -x+0.3139*log(x)+1, 0.5)
```

to get $S(+\infty) = 0.0482 \approx 0.048$ which correspond the $1526 \times 0.048 = 73.248 \approx 73$. So the fraction of the population that contracts the disease is approximately $1 - 0.048 = 0.952$. In other words, the total number of infected residents is equal to $1526 \times 0.952 \approx 1453 = 1526 - 73$.

3. If 500 residents had been vaccinated before the disease started, how many people would be infected during the epidemic?





Solution: Use the initial condition $(S(0), I(0)) = (\frac{1524-500}{1526}, \frac{2}{1526})$ as the new initial condition and re-run the code. We get:

$$\lim_{t \to +\infty} S(t) \approx 0.1135 \approx 0.11$$

which correspond to $1526 \times 0.11 = 167.86 \approx 168$. So the total number of infected residents is equal to $(1526 - 500) - 168 = 858$.

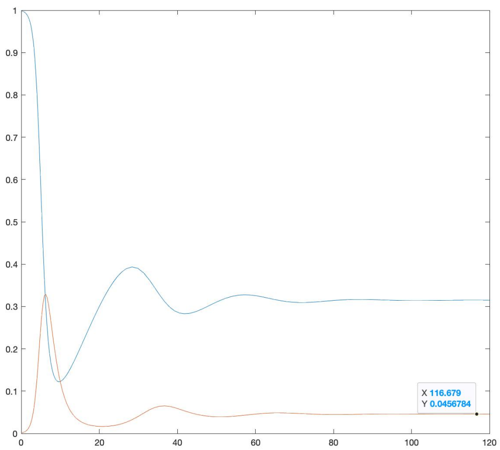
4. Assume that a new strain emerges that can infect those who have recovered from the previous strain. For each unit time, assume 4 percents of recovered people become susceptible again (see Section 2.7, Problem 6). By using the same α, β obtained from above, draw the solution curves and determine what is the fraction of the population that will remain infected as time goes to $+\infty$.

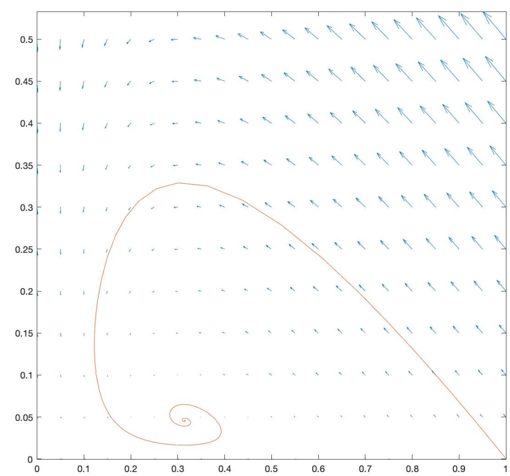
Solution: We need to modify the system as the following (see Section 2.7, Problem 6):

$$\begin{cases} \frac{d}{dt}S(t) = -\alpha IS + \gamma(1 - S - I) \\ \frac{d}{dt}I(t) = \alpha IS - \beta I. \end{cases}$$

Change the parameter $c = \gamma$ from 0 to 0.04 and re-run the MATLAB code. We see that there is a spiraling sink at an equilibrium point $(S_0, I_0) \neq (1, 0)$ satisfying:

$$\begin{cases} -\alpha I_0 S_0 + \gamma(1 - S_0 - I_0) = 0 \\ \alpha I_0 S_0 - \beta I_0 = 0. \end{cases}$$





We solve this to get:

$$S_0 = \frac{\beta}{\alpha}, \quad I_0 = \frac{\gamma(1 - S_0)}{\alpha S_0 + \gamma} = \frac{\gamma(\alpha - \beta)}{\alpha(\beta + \gamma)}.$$

Substituting $\alpha = 1.78$, $\beta = 0.56$ and $\gamma = 0.04$ into the expression we get $S_0 = 0.31$ and $I_0 = 0.0457 \approx 0.046$. So there is about 4.6 percents of population (or $1526 \times 0.046 \approx 70$ people) that will remain infected.