MAT 252 Fall 2021 FINAL EXAM

NAME: **ID:**

THERE ARE SIX (6) PROBLEMS. THEY HAVE THE INDICATED VALUE.

SHOW YOUR WORK

NO CALCULATORS NO CELLS ETC.

ON YOUR DESK: ONLY test, pen, pencil, eraser.

1		16pts
2		16pts
3		16pts
4		16pts
5		16pts
6		20pts
Total		100pts

!!! WRITE YOUR NAME, STUDENT ID. BELOW !!!

NAME : **ID :**

1(16pts) Consider a logistic model with harvesting:

$$\frac{dP}{dt} = P(6 - P) - h.$$

Assume that the initial population is $P(0) = 4$. Solve the following problems:

(**a**) If $h = 9$, find the value of P at $t = 4$.

(**b**) If $h = 5$, what is the population as $t \to +\infty$? Explain your conclusion.

2(16pts) Assume that a huge tank initially contains 4 gallons of salty water with a concentration 1/4 pound/gallon (p/g). Assume that 2 gallons of salty water with a concentration 1/2 p/g flows into the tank at each minute. At the same time, 1 gallon of well-mixed salty water flows out of the tank at each minute.

$$\left(\text{note: concentration} = \frac{\text{amount of salt}}{\text{volume of salty water}}\right)$$

(1) Write down and solve the initial value problem for the amount of salt in the tank.

(2) What is the **concentration** of the salty water in the tank at $t = 6$ min?

3(16pts) Consider the following equation for a forced oscillation :

$$\frac{d^2y}{dt^2} + y = 2\sin(t).$$

Solve the initial value problem with $y(0) = y'(0) = 1$. Is the amplitude of the oscillation bounded as $t \to +\infty$? What phenomenon do we have in this case?

4(16 pts) Consider the linear system with a parameter:

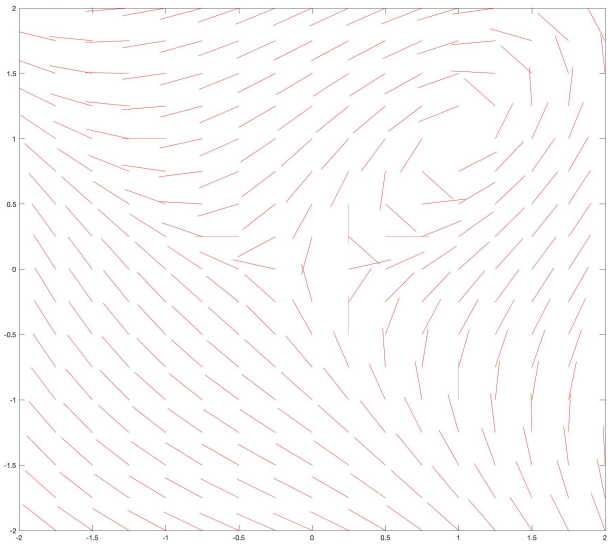
$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -2 & 1 \\ a & 0 \end{pmatrix} \mathbf{Y}.$$

(**a**) When $a = -1$, solve the linear system with initial value $\mathbf{Y}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

(**b**) For what values of a do we have bifurcations? Explain how the type of equilibrium point changes.

5(16pts) Solve the initial value problem:

$$\frac{d\mathbf{Y}}{dt} = \mathbf{AY} \quad \text{with} \quad \mathbf{A} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{pmatrix}, \quad \mathbf{Y}(0) = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}.$$



6(20pts) Consider the non-linear system:

$$\begin{cases} \frac{dx}{dt} = x - y^2 \\ \frac{dy}{dt} = -y + x^2. \end{cases}$$

(1) Find all equilibrium points. Calculate the linearization at each of them and classify the types of equilibrium points for the linearized systems.
(2) Is the above nonlinear system a Hamiltonian system? If yes, find a Hamiltonian function.
(3) What are the types for equilibrium points for the nonlinear system? Explain your reason.
(4) The above attached picture shows the directional field in the phase plane but misses arrow heads. Find the correct directions of the vectors and add arrow heads to the picture to show the correct flow directions of solution curves. Always explain your works.

Continuation of Problem 6:

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Extra page