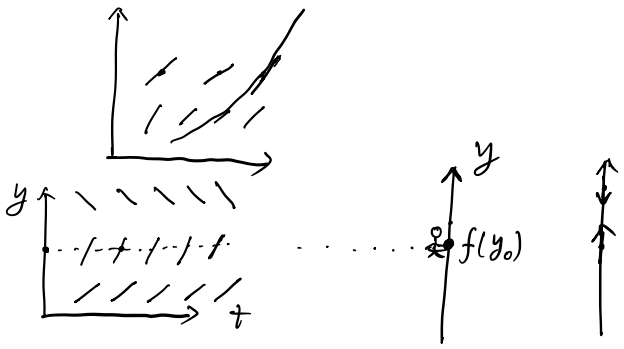
$\frac{dy}{dt} = f(y,t)$

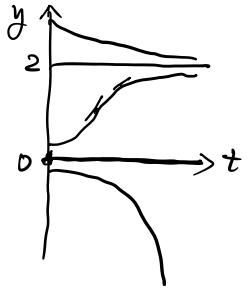
$\frac{dy}{dt} = f(y)$

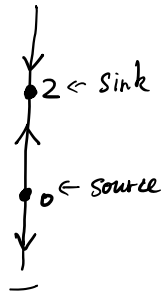


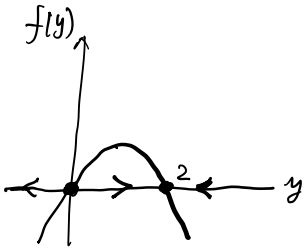
phase line

Ex: Logistic model $\frac{dy}{dt} = k \cdot y \cdot (1 - \frac{y}{N})$ $\quad -(y-1)^2+1$

$k=2,\ N=2.\quad \frac{dy}{dt} = 2 \cdot y \cdot (1 - \frac{y}{2}) = 2y - y^2 = f(y)$

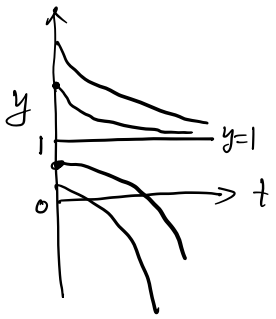


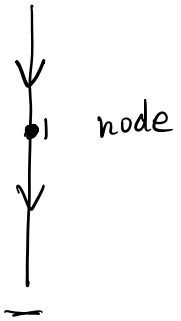


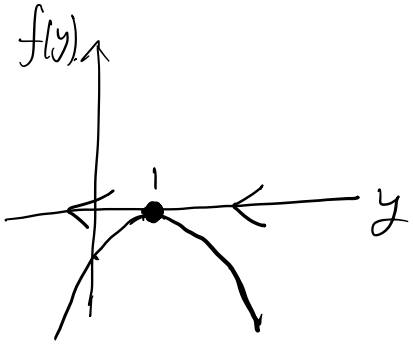


Ex: Logistic model with harvesting $\frac{dy}{dt} = 2y - y^2 - h$ ← parameter

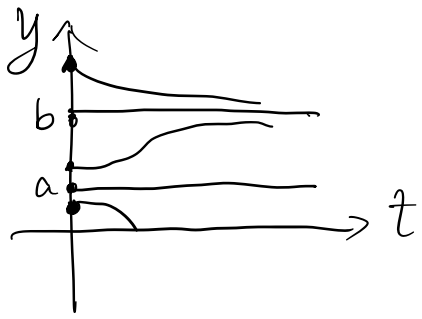
$h=1:\quad f(y) = -y^2+2y-1 = \underline{-(y-1)^2}.$

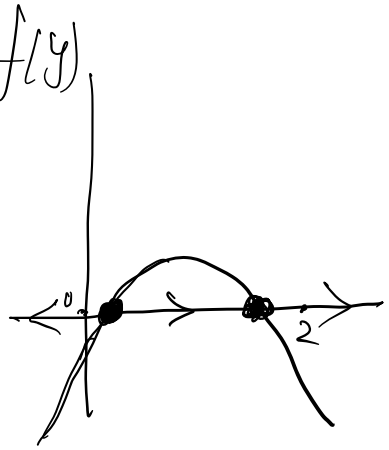




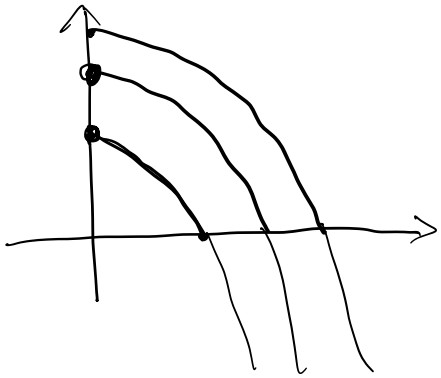


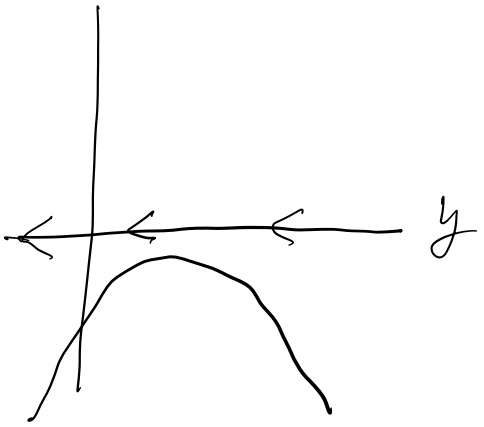
Ex: $0 < h < 1$



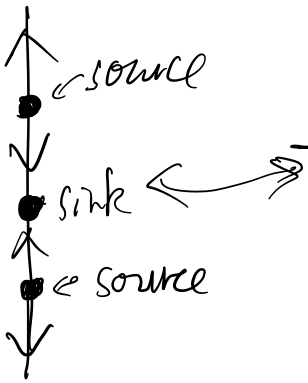


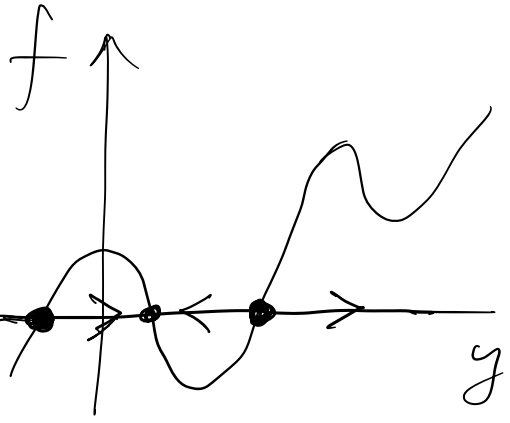
Ex: $h > 1$

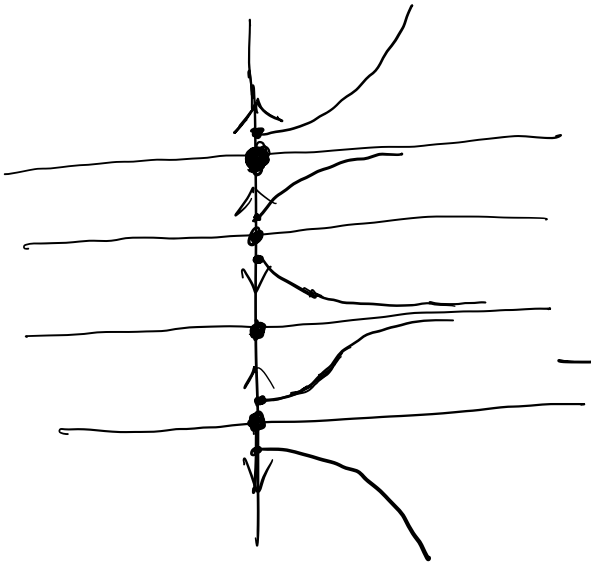


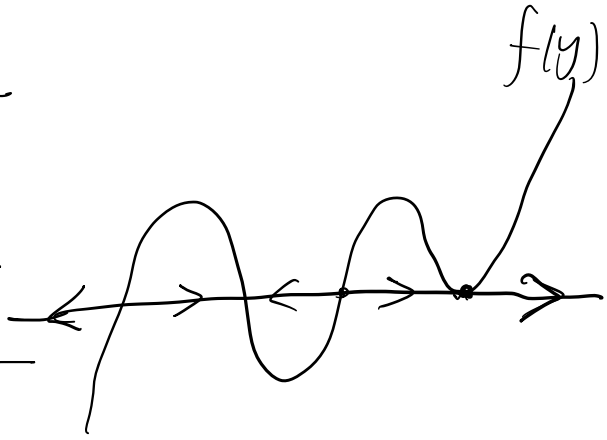


Ex: $\frac{dy}{dt} = f(y)$

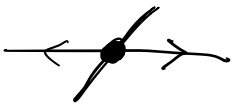
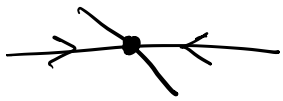
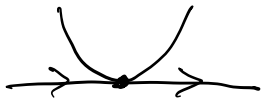




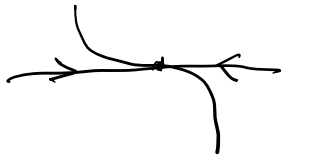


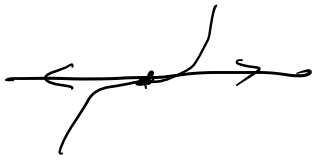


$f(y)$. $f(y_0)=0 \Rightarrow y_0$ is a equilibrium value

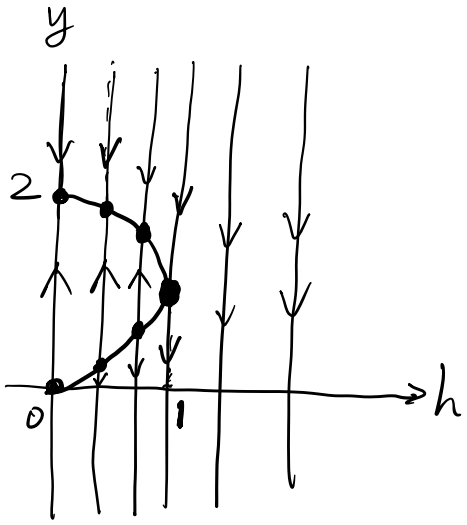
- $f'(y_0)>0$  y_0 is source
- $f'(y_0)<0$  y_0 is a sink.
- $f'(y_0)=0$  node

no conclusion in general.

 Sink

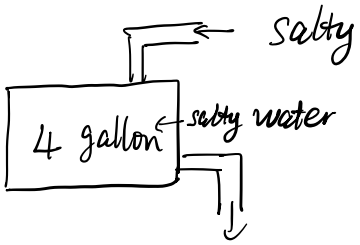
 source

Ex: $\frac{dy}{dt} = 2y - y^2 - h$



bifurcation diagram.

Quiz: Mixing Problem



- salty water · 2 gallon per minute
- concentration = 0.5 pounds/gallon

Initial salt: 3 pounds in the tank.

Question: 1. Find the amount of salt at time t in the tank.

2. What is the amount of salt in the tank as $t \to +\infty$?