• Logistic model: $\frac{dP}{dt} = k\left(1-\frac{P}{N}\right)\cdot P = \frac{k}{N}(N-P)\cdot P$

$$\Rightarrow \frac{N}{(N-P)P}\,dP = k\,dt \Rightarrow \int \frac{N}{(N-P)P}\,dP = \int k\,dt = kt + C_1$$

$$\frac{N}{(N-P)P} = \frac{1}{N-P} + \frac{1}{P} \qquad \int\left(\frac{1}{N-P}+\frac{1}{P}\right)dP = -\ln|N-P| + \ln|P| = \ln\left|\frac{P}{N-P}\right|$$

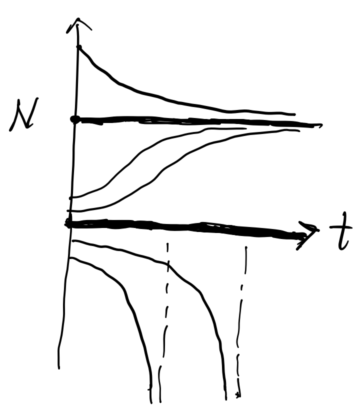
$$\Rightarrow \ln\left|\frac{P}{N-P}\right| = kt + C_1 \Rightarrow \left|\frac{P}{N-P}\right| = e^{C_1}\cdot e^{kt} = C_2\cdot e^{kt} \quad C_2>0$$

$$\Rightarrow \frac{P}{N-P} = C_2 e^{kt} \quad C_2 \in \mathbb{R} \Rightarrow P = C_2\cdot e^{kt}(N-P) = C_2\cdot e^{kt}N - C_2\cdot e^{kt}\cdot P$$

$$\Rightarrow (1+C_2\cdot e^{kt})P = C_2\cdot e^{kt}\cdot N \Rightarrow P = P(t) = \frac{C_2\cdot e^{kt}\cdot N}{1+C_2\cdot e^{kt}}$$ general solution.

• $P(0) = \frac{C_2\cdot N}{1+C_2} = P_0 \Rightarrow C_2 N = P_0 + C_2\cdot P_0 \Rightarrow C_2 = \frac{P_0}{N-P_0}$

$$\Rightarrow P(t) = \frac{\frac{P_0}{N-P_0}\cdot e^{kt}\cdot N}{1+\frac{P_0}{N-P_0}e^{kt}} = \frac{P_0\cdot e^{kt}\cdot N}{(N-P_0)+P_0 e^{kt}}$$ solution to $\begin{cases} \frac{dP}{dt} = k\left(1-\frac{P}{N}\right)P \\ P(0) = P_0 \end{cases}$



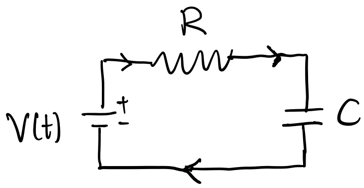
equilibrium solution: • $P \equiv 0 = \frac{0\cdot e^{kt}\cdot N}{(N-0)+0\cdot e^{kt}}$

• $P \equiv N = \frac{N\cdot e^{kt}\cdot N}{(N-N)+N\cdot e^{kt}}$

$$P_0 > 0 \Rightarrow P(t) = \frac{N}{1+\left(\frac{N}{P_0}-1\right)e^{-kt}} \xrightarrow{t\to+\infty} N$$

$$P_0 < 0 \Rightarrow P(t) = \frac{N}{1-\left(1+\frac{N}{|P_0|}\right)e^{-kt}}$$ goes to $-\infty$ when $t \to \frac{1}{k}\log\left(1+\frac{N}{|P_0|}\right)$

- RC circuits

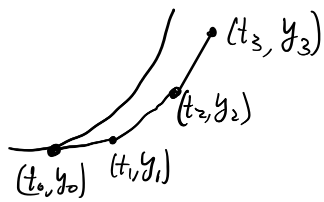


$$\begin{cases} V(t) = \overset{\text{current}}{\overset{\downarrow}{I}} \cdot R + v_c \\ I = C \cdot \dfrac{dv_c}{dt} \end{cases} \Rightarrow V(t) = C \cdot \frac{dv_c}{dt} R + v_c$$

$$\Downarrow$$

$$\frac{dv_c}{dt} = \frac{V(t) - v_c}{C \cdot R}$$

- Euler's method:



Δt: step size

$t_1 = t_0 + \Delta t$, $y_1 = y_0 + f(t_0, y_0)\Delta t$

$t_2 = t_1 + \Delta t$, $y_2 = y_1 + f(t_1, y_1)\Delta t$

......

$t_k = t_{k-1} + \Delta t$, $y_k = y_{k-1} + f(t_{k-1}, y_{k-1})\Delta t$.

<u>Ex</u>: $\dfrac{dy}{dt} = \underbrace{-y + t}_{f(t,y)}$. $y(\underset{t_0}{0}) = \underset{y_0}{0}$, $\underline{\Delta t = 0.5}$, $y(1) = ?$

$t_1 = t_0 + \Delta t = 0.5$, $y_1 = y_0 + f(t_0, y_0)\Delta t = 0 + (-0+0)\,0.5 = 0$

$t_2 = t_1 + \Delta t = 1$, $y_2 = y_1 + f(t_1, y_1)\Delta t = 0 + (-0+0.5) \times 0.5 = 0.25$

If $\Delta t = 0.25$, then: $t_1 = 0.25$, $y_1 = 0 + (-0+0) \cdot 0.25 = 0$.

$t_2 = 0.5$, $y_2 = 0 + (-0 + 0.25) \cdot 0.25 = 0.0625$

$t_3 = 0.75$, $y_3 = 0.0625 + (-0.0625 + 0.5) \cdot 0.25 = 0.171875 \sim 0.17$

$t_4 = 1$, $y_4 \approx 0.17 + (-0.17 + 0.75) \cdot 0.25 = 0.315$

Exact solution: $\frac{dy}{dt} + y = t \Rightarrow \frac{d}{dt}(e^t y) = t e^t$

$\Rightarrow \quad e^t y = \int t \cdot e^t dt = \int t \cdot de^t = t \cdot e^t - e^t + C$

$\Rightarrow \quad y(t) = t - 1 + C \cdot e^{-t}$. $\quad y(0) = -1 + C = 0 \Rightarrow C = 1$

$\Rightarrow \quad y(t) = t - 1 + e^{-t}$, $\quad y(1) = e^{-1} \sim 0.368$.

$\left(\frac{dy}{dt} = 1 - e^{-t} = -y + t\right)$