- variables — dependent (functions) : y
  — independent (time) : t

$$y(t) = \int f(s)\,ds$$

- rule $\quad y' = f(t, y)$ $\quad$ ex: $y' = 2y$, $y' = f(t)$

$$y' = \frac{dy}{dt}$$

$$y' = g(y)$$

autonomous : rule does not change over time.

- initial condition $\begin{cases} y' = f(t, y) \\ y(0) = y_0 \end{cases} \rightsquigarrow y = y(t)$

Ex: of initial value problem $\boxed{\begin{cases} y' = f(t) \\ y(0) = y_0 \end{cases}} \to y(t) = \int_0^t y'(s)\,ds + y(0) = \int_0^t f(s)\,ds + y_0$

$$\left(y(t) - y(0) = \int_0^t y'(s)\,ds \right)$$

- Population model. $\quad$ - P = population $\quad t$ = time.
  - rule: growth rate of P is proportional to P.

$$\frac{dP}{dt} = k \cdot P \qquad k \text{ is a parameter.}$$

  - initial population $P(0) = P_0$

Ex:

IVP: $\begin{cases} \frac{dP}{dt} = k \cdot P \text{ (autonomous)}. \\ P(0) = P_0 \end{cases}$ $\quad \boxed{P(t) = e^{kt}}$ is a solution

$$\frac{dP}{dt} = e^{kt} \cdot k = k \cdot P$$

linear ODE $\leftarrow$ $\boxed{\text{ordinary differential equation}}$

$\boxed{P(t) = C \cdot e^{kt} \text{ is a solution for any } C \in \mathbb{R}.}$

real number.

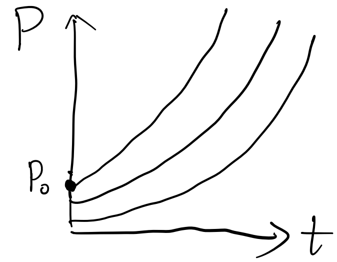
$$P_0 = P(0) = C e^{k \cdot 0} = C \cdot 1 = C$$

$$\frac{dP}{dt} = C e^{kt} \cdot k = k \cdot P.$$

$$C e^{kt} \cdot k = k \cdot (C e^{kt})$$

$\rightsquigarrow \left(\underline{P(t) = P_0 \cdot e^{kt}}, \text{ for } t \geq 0. \text{ is the solution to the IVP.} \right)$

(initial value problem)



$$\frac{dP}{dt} = k\, P$$

relative growth rate

- Logistic model (with more realistic assumption (rule))

assumption: the relative growth rate decreases as P increases.

$$\frac{dP}{dt} = k\cdot\left(1-\frac{P}{N}\right) P$$

(k, N: parameters)

N: Capacity

$\begin{pmatrix} k=4 \\ N=2 \end{pmatrix}$

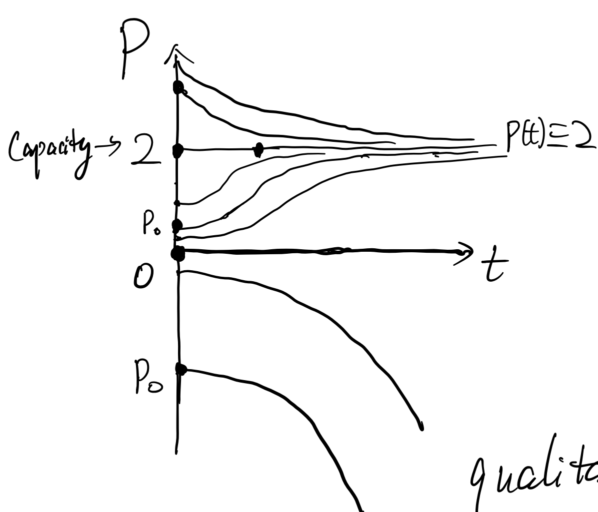
$$\begin{cases} \dfrac{dP}{dt} = 4\left(1-\dfrac{P}{2}\right)\cdot P \\ P(0) = 0 \end{cases}$$

$\leftarrow$ $P(t) \equiv 0$ is the solution

equilibrium solutions.

$$\begin{cases} \dfrac{dP}{dt} = 4\cdot\left(1-\dfrac{P}{2}\right)\cdot P \\ P(0) = 2. \end{cases}$$

$\leftarrow$ $P(t) \equiv 2$ is the solution



$P_0 > 2$, $\left.\dfrac{dP}{dt}\right|_{t=0} = 4\underbrace{\left(1-\dfrac{P_0}{2}\right)}_{<0}\cdot P_0 < 0.$

$0 < P_0 = P(0) < 2$. $\left.\dfrac{dP}{dt}\right|_{t=0} = 4\left(1-\dfrac{P_0}{2}\right)\cdot P_0 > 0.$

$P_0 < 0$, $\dfrac{dP}{dt} = 4\underbrace{\left(1-\dfrac{P_0}{2}\right)}_{>0}\cdot \underbrace{P_0}_{<0} < 0.$

qualitative behavior.

$$\frac{dy}{dt} = t - y^2$$

analytic

qualitative

numerical