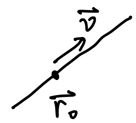
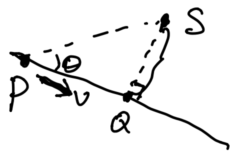
12.3 lines



$$\vec r(t) = \vec r_0 + t\cdot\vec v = \vec r_0 + t\cdot\underbrace{|\vec v|}_{\text{speed}}\cdot\underbrace{\frac{\vec v}{|\vec v|}}_{\text{direction}}$$

$$\Updownarrow$$

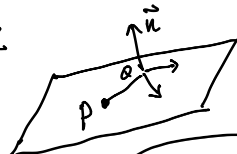
$$x(t) = x_0 + t\cdot v_1,\quad y(t) = y_0 + t v_2,\quad z(t) = z_0 + t v_3.$$



$$|\vec{SQ}| = |\vec{PS}|\cdot\sin\theta = |\vec{PS}|\cdot\underbrace{\left|\frac{\vec v}{|\vec v|}\right|}_{=1}\cdot\sin\theta$$

distance from S to the line $= \left|\vec{PS}\times\frac{\vec v}{|\vec v|}\right| = \dfrac{|\vec{PS}\times\vec v|}{|\vec v|}.$

planes



$= \{Q\in\mathbb{R}^3 ;\ \vec{PQ}\perp\vec n\}$ $\quad \vec{PQ}\cdot\vec n = 0$

set of triples in 3-dim space pts

$\langle A, B, C\rangle = \vec n$

$$\boxed{Ax+By+Cz=D}$$

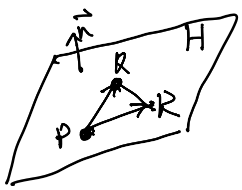
$\vec n = \langle A, B, C\rangle$ $\quad P(x_0, y_0, z_0)\quad Q(x,y,z)$

$\langle x-x_0, y-y_0, z-z_0\rangle\cdot\langle A, B, C\rangle = 0$

$$\boxed{A(x-x_0)+B(y-y_0)+C(z-z_0)=0} \iff Ax+By+Cz = \overbrace{Ax_0+By_0+Cz_0}^{D}$$

Ex: $x-2y+3z=4 \Rightarrow \vec n = \langle 1,-2,3\rangle \perp$ plane

Ex:



$P=\langle 1,-2,1\rangle$, $Q=\langle 2,1,3\rangle$, $R=\langle -2,3,1\rangle$

$\vec{PQ} = \langle 1,3,2\rangle$, $\vec{PR} = \langle -3,5,0\rangle$

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \vec i & \vec j & \vec k \\ 1 & 3 & 2 \\ -3 & 5 & 0\end{vmatrix} = \vec i(-10) - \vec j\,(6) + \vec k\cdot \underset{5+9}{14} = \langle -10,-6,14\rangle \checkmark$$

$\langle -10,-6,14\rangle\cdot\langle 1,3,2\rangle = -10-18+28=0$

$30-30+0=0$

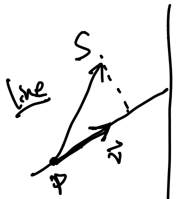
H: $\overset{5}{\cancel{-10}}\cdot(x-2) - \overset{3}{\cancel{6}}(y-1) + \overset{7}{\cancel{14}}(z-3) = 0$

$$\Updownarrow$$

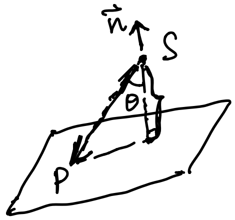
$-5x-3y+7z+10+3-21=0 \quad \boxed{5x+3y-7z=-8}$

$5-6-7=-8$

distance from a point to a plane



$$\frac{|\vec{PS}\times\vec{v}|}{|\vec{v}|}$$

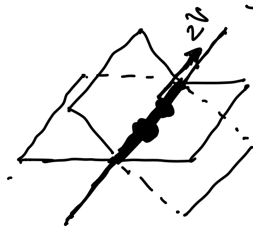


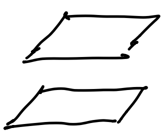
$$\text{distance} = |\vec{SP}|\cdot|\cos\theta| = |\vec{SP}|\underbrace{\left|\frac{\vec{n}}{|\vec{n}|}\right|}_{=1}|\cos\theta|$$

$$= \left|\vec{SP}\cdot\frac{\vec{n}}{|\vec{n}|}\right| = \left|\mathrm{Proj}_{\vec{n}}\vec{SP}\right| = \frac{|\vec{PS}\cdot\vec{n}|}{|\vec{n}|}$$

H_1: $\vec{n}_1\cdot\langle x-x_0, y-y_0, z-z_0\rangle = 0$

H_2: $\vec{n}_2\cdot\langle x-a_0, y-b_0, z-c_0\rangle = 0$.





$\vec{v}\perp\vec{n}_1$, $\vec{v}\perp\vec{n}_2$ $\Rightarrow$ choose $\vec{v}=\vec{n}_1\times\vec{n}_2$ (nonzero if and only if $\vec{n}_1 \not\parallel \vec{n}_2$)

$2x-4y=2$

$\begin{cases} x-2y=1 \\ 2x+3y=2 \end{cases}$

$7y=0 \Rightarrow y=0$

Ex: l: $\begin{cases} x-2y+z=1 \\ 2x+3y-4z=2 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 2 & 3 & -4 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 7 & -6 & 0 \end{array}\right)$$

free variable.

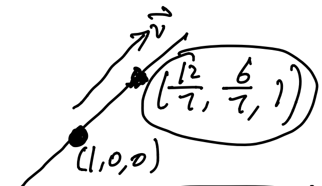
$z=1$, $y=\frac{6}{7} \Rightarrow x = 1+2y-z$

$2\times\frac{12}{7}+3\times\frac{6}{7}-4\times 1 = \frac{24+18}{7}-4 = 6-4=2$ ✓ $\quad = 1+\frac{12}{7}-1 = \frac{12}{7}$

$z=0$: $y=0 \Rightarrow x=1$. $(1,0,0)\in l$ $\quad \boxed{\left(\frac{12}{7}, \frac{6}{7}, 1\right)}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ 2 & 3 & -4 \end{vmatrix} = \vec{i}\cdot 5 - \vec{j}\cdot(-4-2) + \vec{k}\cdot(3+4) = \langle 5, 6, 7\rangle = \vec{v}$$

$(-2)\times(-4)-3=5$



line: $\boxed{x(t)=1+5t,\ y(t)=6t,\ z(t)=7t.}$

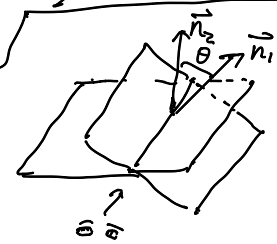
$\boxed{\left(\frac{12}{7}+5t,\ \frac{6}{7}+6t,\ 1+7t\right)}$

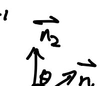
$\vec{n}_1=\langle 1,2,1\rangle$, $\vec{n}_2=\langle 2,3,-4\rangle$

$$\boxed{\theta = \cos^{-1}\left(\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right)}$$

$\vec{n}_1\cdot\vec{n}_2 = |\vec{n}_1||\vec{n}_2|\cos\theta$.

$$\theta = \cos^{-1}\left(\frac{2+6-4}{\sqrt{1+4+1}\sqrt{4+9+16}}\right) = \cos^{-1}\left(\frac{4}{\sqrt{6}\cdot\sqrt{29}}\right)$$





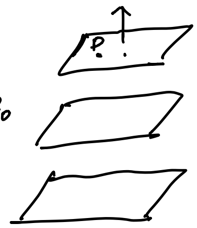
$\vec{n}_1\perp\vec{n}_2 \Leftrightarrow \vec{n}_1\cdot\vec{n}_2=0$

$\boxed{Ax+By+Cz=D}$

$\vec{n}=\langle A,B,C\rangle$

$x + 2y + z = D$

$= Ax_0 + By_0 + Cz_0$



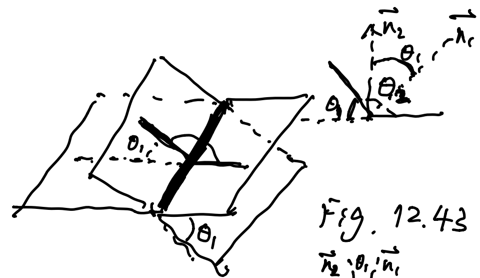
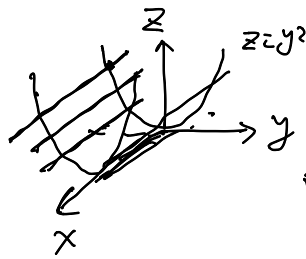


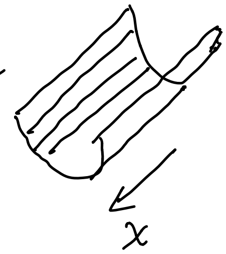
Fig. 12.43

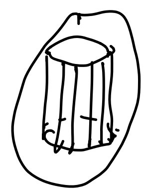
12.6: $z = y^2$

$y^2 - z = 0$

in 3-dim space



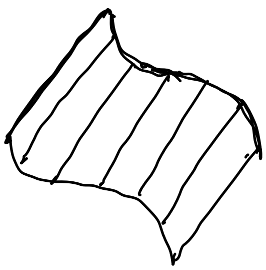




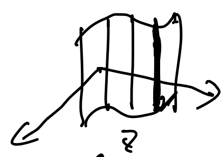
curve $<$ $F(x, y, z) = 0$ ⇝ surface

$G(x, y, z) = 0$.

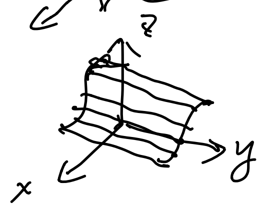
cylinder: surface generated by moving a straight line along a given curve



$\boxed{f(x, y) = 0}$



$f(x, z) = 0$



plane

$Ax + By + Cz = D$

Quadric surfaces: $Ax^2 + By^2 + Cz^2 = E$. ?

Ex: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Fix z: $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{z^2}{c^2}}$

ellipsoid.

$a = b = c$ $\boxed{x^2 + y^2 + z^2 = a^2}$

$x^2 + y^2 + z^2 = 3^2$