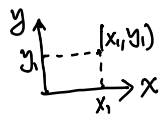
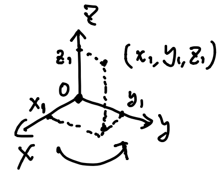
12.1 3-dim Coordinate System

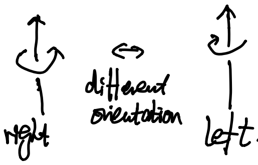
2-dim



3-dim



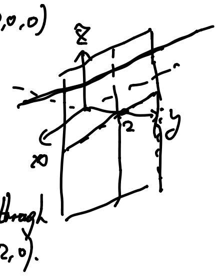
right-handed coordinate



x-axis : $(x, 0, 0)$ xy-plane: $(x, y, 0)$

y-axis : $(0, y, 0)$ yz-plane : $(0, y, z)$ origin $(0,0,0)$

z-axis : $(0, 0, z)$ xz-plane: $(x, 0, z)$



Ex: $y=2$ plane consisting of pts whose y-coordinate $=2$

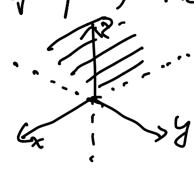
$=$ plane that is perpendicular to the y-axis passing through $(0,2,0)$.

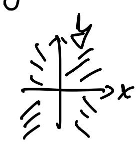
$y=2, z=3$: pts of form $(x, 2, 3)$.

$=$ line of intersection $\{y=2\} \cap \{z=3\}$.

$=$ line that is parallel to the x-axis passing through $(0, 2, 3)$.

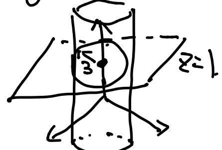
$z \geq 0$: pts above the xy-plane (upper half space) including xy-plane

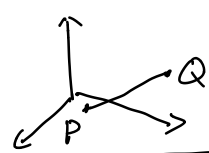


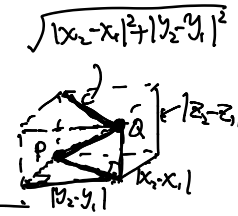


$\boxed{x \leq 0, y \leq 0, z \geq 0}$ $\boxed{\text{Octant}}$ regions splitted by 3 coordinate planes

$\boxed{x^2+y^2=9, z=1}$



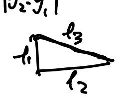




Distances $P = (x_1, y_1, z_1)$

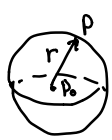
$Q = (x_2, y_2, z_2)$

$$|\overline{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$



$l_1^2 + l_2^2 = l_3^2$

Spheres



$P_0 = (x_0, y_0, z_0)$. $P = (x, y, z)$

$$|\overline{PP_0}| = r \Leftrightarrow |\overline{PP_0}|^2 = r^2$$

$$\boxed{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = r^2}$$ ← standard form

$\Updownarrow$

$$x^2+y^2+z^2 - 2x_0x - 2y_0y - 2z_0z = r^2 - x_0^2 - y_0^2 - z_0^2$$

Ex: $2x^2+2y^2+2z^2-4x+6y-5z=1$ center $= (1, -\frac{3}{2}, \frac{5}{4})$ $r = \frac{\sqrt{85}}{4}$

$24+61=85$

$x^2-2x + y^2+3y + z^2 - \frac{5}{2}z = \frac{1}{2}$

$$\underbrace{(x^2-2x+1)}_{(x-1)^2} + \underbrace{y^2+3y+(\tfrac{3}{2})^2}_{(y+\frac{3}{2})^2} + \underbrace{z^2-\tfrac{5}{2}z+(\tfrac{5}{4})^2}_{(z-\frac{5}{4})^2} = \frac{1}{2}+1+\frac{9}{4}+\frac{25}{16} = \frac{8+16+36+25}{16} = r^2$$

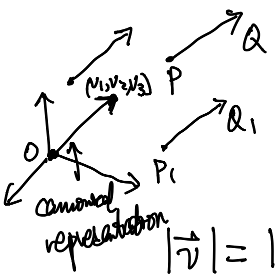
Ex: $\underbrace{x^2+y^2+z^2}_{\overset{\parallel}{|\vec{PO}|^2}} < 4$: pts. whose distance to origin < 2.

$\qquad$ = strictly inside the sphere 

≤ 4 :

> 4 : outside the sphere

12.2 Vectors : describe velocity, force, displacement.



$\vec{v} = \vec{PQ} = \vec{P_1Q_1}$ if $\begin{cases} \text{same direction} \\ |\vec{PQ}| = |\vec{P_1Q_1}| \end{cases}$

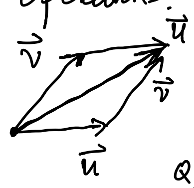
$\| \ \langle v_1, v_2, v_3 \rangle$ $\qquad P = (x_1, y_1, z_1)$

$\| \ \langle x_2-x_1, y_2-y_1, z_2-z_1 \rangle$ $\qquad Q = (x_2, y_2, z_2)$

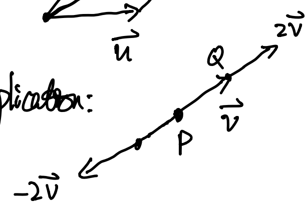
$|\vec{v}| = |\vec{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$ length / magnitude of $\vec{v}$

$\vec{0} = \langle 0,0,0 \rangle$ zero vector

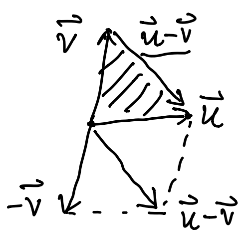
Vector algebra Operations:

addition: 

parallelogram law

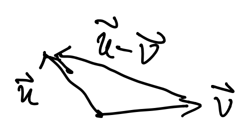
Scalar multiplication: 

$0 \cdot \vec{v} = \vec{0}$ $\qquad \underset{\text{scalar}}{k} \cdot \underset{\text{vector}}{\vec{v}}$

subtract: 

$\vec{u}-\vec{v} = \vec{u} + (-\vec{v})$

$\vec{v} + (\vec{u}-\vec{v}) = \vec{u}$



$\vec{u} = \langle u_1, u_2, u_3 \rangle$ $\qquad \vec{u} \pm \vec{v} = \langle u_1 \pm v_1, u_2 \pm v_2, u_3 \pm v_3 \rangle$

$\vec{v} = \langle v_1, v_2, v_3 \rangle$ $\qquad k \cdot \vec{u} = \langle k u_1, k u_2, k u_3 \rangle$

$\vec{u} = \langle 2, -1, 1 \rangle$ $\qquad 3\vec{u} - 5\vec{v} = 3\langle 2,-1,1 \rangle - 5\langle 0,1,3 \rangle$

$\vec{v} = \langle 0, 1, 3 \rangle$ $\qquad = \langle 6,-3,3 \rangle - \langle 0,5,15 \rangle$

$\qquad = \langle 6, -8, -12 \rangle$

$$(\vec{v}_1+\vec{v}_2)+\vec{v}_3 = \vec{v}_1+(\vec{v}_3+\vec{v}_2)$$

associativity $\parallel$ $\qquad$ commutativity

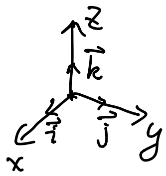
$$\vec{v}_1+(\vec{v}_2+\vec{v}_3)$$

$$\underset{\text{scalar}}{k}\cdot \underset{\text{vector}}{(\vec{v}_1+\vec{v}_2)} = k\vec{v}_1 + k\vec{v}_2 \quad \text{distributivity.}$$

$\boxed{\text{2-dim}}$ $\to$ $\vec{v} = \langle v_1, v_2 \rangle$ $\quad \vec{v}-\vec{u} = \langle v_1-u_1, v_2-u_2 \rangle$

$\vec{u} = \langle u_1, u_2 \rangle$

$\vec{v}$ is a Unit vector if $|\vec{v}|=1$.

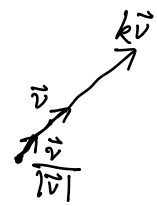


Ex: $\vec{i} = \langle 1,0,0 \rangle$, $\vec{j} = \langle 0,1,0 \rangle$, $\vec{k} = \langle 0,0,1 \rangle$

$$\vec{v} = \langle v_1, v_2, v_3 \rangle = v_1 \langle 1,0,0 \rangle + v_2 \langle 0,1,0 \rangle + v_3 \langle 0,0,1 \rangle$$

$$\vec{v} = v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k}$$

$$|k\vec{v}| = |k|\,|\vec{v}|$$



$\vec{0} \neq \vec{v}$

$\Updownarrow$

$|\vec{v}| \neq 0$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{1}{|\vec{v}|}\cdot\vec{v} \quad \boxed{\text{unit vector}} \quad \left|\frac{\vec{v}}{|\vec{v}|}\right| = \frac{1}{|\vec{v}|}\cdot|\vec{v}| = 1$$

scalar multiplication

unit vector in the direction of $\vec{v}$.