17. Implicit Differentiation $F(x,y)=0 \rightsquigarrow \underline{y=y(x)}$.

$$\boxed{2x^2-2x\cdot y+3y^2=60}$$

$\frac{dy}{dx}=?$

$$\frac{d}{dx}(2x^2-2xy+3y^2)=\underline{\frac{d}{dx}60=0}$$

$\frac{d}{dx}(fg)=f'\cdot g+f\cdot g'$

$$\| $$

$$4x-2\underline{\frac{d}{dx}(x\cdot y)}+3\cdot\underline{\frac{d}{dx}(y^2)}$$

$$\| \qquad\qquad \|$$

$$\underline{4x-2\left(1\cdot y+x\cdot\frac{dy}{dx}\right)+3\cdot 2\cdot y\cdot\frac{dy}{dx}}$$

$$\|$$

$$(4x-2y)+(-2x+6y)\cdot\frac{dy}{dx}=0$$

$$\Rightarrow (4x-2y)=(2x-6y)\cdot\frac{dy}{dx}$$

(a) $\Rightarrow \frac{dy}{dx}=\frac{4x-2y}{2x-6y}=\frac{2x-y}{x-3y}$

(b) Set $2x-y=0 \Rightarrow y=2x$

$$2x^2-2x\cdot 2x+3\cdot(2x)^2=60 \Rightarrow x^2=\frac{60}{10}=6$$

$$\| \qquad\qquad \|$$

$$(2-4+12)\cdot x^2=10x^2 \qquad \Rightarrow \underline{x=\pm\sqrt{6}}$$

$y=2x=\pm2\sqrt{6}$

$$\underline{(\sqrt{6},2\sqrt{6}),\ (-\sqrt{6},-2\sqrt{6})}$$

18: (a) $\int_2^4 \frac{18t-3t^2}{t}\,dt$

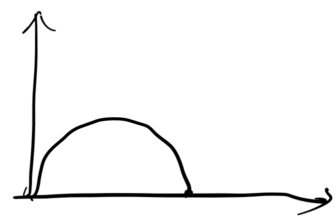
$= \int_2^4 (18-3t)\,dt = \left(18t - \frac{3}{2}t^2\right)\Big|_2^4$

$= \left(18\cdot 4 - \frac{3}{2}4^2\right) - \left(18\cdot 2 - \frac{3}{2}2^2\right)$

$= (72-24) - (36-6) = 48-30 = 18.$

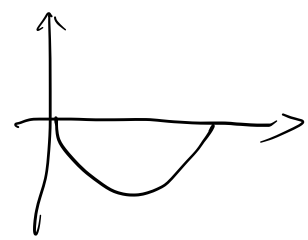
(b)

$$\int_0^{\frac{\pi}{2}} 23\sin(x)\underset{\underset{(\cos x)^2}{\parallel}}{\cos^2(x)}\,dx$$



- $\underline{u = \cos x}$, $du = -\sin x \cdot dx$

$dx = \frac{du}{-\sin x}$



$$\int 23 \cdot \cancel{\sin x} \cdot u^2 \cdot \frac{du}{-\cancel{\sin x}}$$

$$-23\int_{\underset{\underset{\cos(0)}{\parallel}}{1}}^{\overset{\overset{\cos(\frac{\pi}{2})}{\parallel}}{0}} u^2\,du = -23\cdot\frac{1}{3}u^3\Big|_1^0 = -23\cdot\frac{1}{3}(0^3-1^3) = \frac{23}{3}$$

$\parallel$

$23\int_0^1 u^2\,du$

$$\boxed{\int_0^{\frac{\pi}{2}} \underset{\underset{(\sin x)^3}{\parallel}}{\sin^3 x} \cdot \underset{\underset{(\cos x)^2}{\parallel}}{\cos^2 x}\, dx}$$

$\boxed{u = \cos x}$. $\quad du = -\sin x \cdot dx \Rightarrow \quad dx = -\dfrac{du}{\sin x}$

$$\int \sin^3 x \cdot u^2 \cdot \left(-\frac{du}{\sin x}\right) = \int \underset{\underset{(1-u^2)}{\parallel}}{\sin^2 x}\, u^2 (-du).$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \underset{\underset{u^2}{\parallel}}{\cos^2 x}$

$$-\int_1^0 (1-u^2)\cdot u^2\, du$$

$\parallel$

$$\int_0^1 (u^2 - u^4)\, du = \left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)\Big|_0^1$$

19. (a) $\frac{1-1}{\ln(1)} = \frac{0}{0}$

$$\lim_{x\to 1} \frac{4x^3-1}{\frac{1}{77x-76}\cdot 77} = \frac{4-1}{\frac{77}{77-76}} = \frac{3}{77}$$

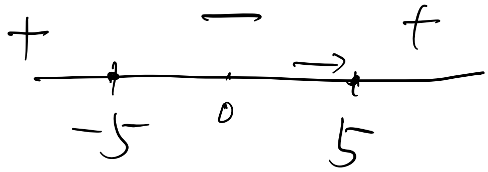
(b)
$$\lim_{x\to-\infty} \frac{\sqrt{x^2\left(36+\frac{63}{x^2}\right)}}{31\cdot x} = \lim_{x\to-\infty} \frac{\overset{-\cancel{x}}{\overset{\shortparallel}{|x|}}\sqrt{36+\frac{63}{x^2}}}{31\cdot \cancel{x}}$$

$$= -\lim_{x\to-\infty} \frac{\sqrt{36+\frac{63}{x^2}}}{31} = -\frac{\sqrt{36}}{31} = -\frac{6}{31}$$

(c)
$$\underline{\lim_{x\to 2^+} f(x)} = \lim_{x\to 2^+} \frac{\overset{(x+2)(x-2)}{x^2-4}}{x-2} = \lim_{x\to 2^+} (\cancel{x}+2) = 4$$

$$f(x) = \begin{cases} 1+4x & x < 2 \\ 8 & x = 2 \\ \frac{x^2-4}{x-2} & x > 2 \end{cases} \qquad f(2) = 8$$

(d) $\lim_{x \to 5^-} \frac{\cos(\pi x)}{x^2-25} = \frac{\cos(5\pi)}{5^2-25} = \frac{-1}{0^-} = +\infty$



L'Hopital only for indeterminate form:

$\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty$

20: $a(t) = \frac{d}{dt}v(t) = 1 + \frac{3}{\sqrt{t}}$

$v(1) = -20$, $v(16) = ?$

$$v(t) = \int \left(1 + \frac{3}{\sqrt{t}}\right) dt = t + 3\cdot\frac{1}{-\frac{1}{2}+1} t^{-\frac{1}{2}+1} + C$$

$$\frac{3}{\sqrt{t}} = 3\cdot t^{-\frac{1}{2}}$$

$$= t + 3\cdot\frac{1}{\frac{1}{2}}\cdot t^{\frac{1}{2}} + C$$

$$\int t^p\, dt = \frac{1}{p+1} t^{p+1} + C$$

$$= t + 6\cdot t^{\frac{1}{2}} + C$$

$-20 = v(1) = 1 + 6\cdot 1^{\frac{1}{2}} + C = 7 + C$

$\Rightarrow \; C = -20 - 7 = -27$

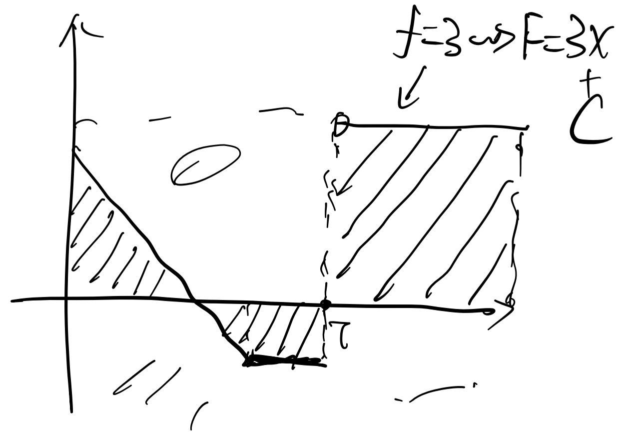
$\Rightarrow \; v(t) = t + 6\cdot t^{\frac{1}{2}} - 27$

$\Rightarrow \; v(16) = 16 + 6\cdot 16^{\frac{1}{2}} - 27 = 16 + 6\cdot 4 - 27$

$= 16 + 24 - 27 = 40 - 27 = 13$

21. $F(x) = \int_0^x f(t)\,dt$

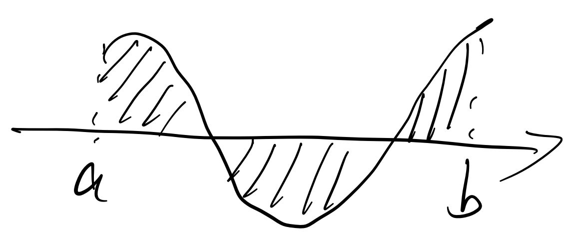
= net area



$\frac{1}{2}\cdot 2\cdot 4 - \Big(\underbrace{\frac{1}{2}\cdot 1\cdot 2}_{1} + \underbrace{1^2}_{\frac{1}{2}(3+1)\cdot 1}\Big) + 3\cdot 3$

$\|$

$4 - (2) + 9 = 11$



$\int_a^b f(x)\,dx$

(b) $F'(6) = f(6) = -1.$

$$\boxed{F(x) = \int_0^x f(t)\,dt \qquad \frac{dF}{dx} = f(x)}$$

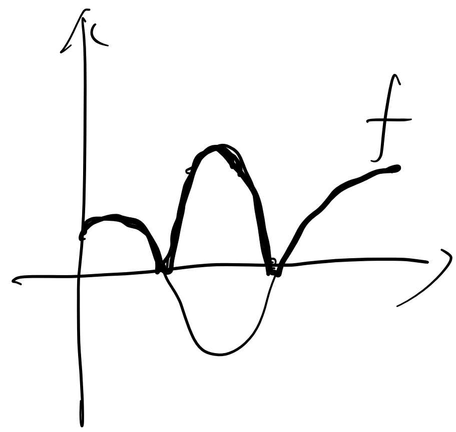
(c) $\int_0^6 |f(t)|\,dt$

$\quad\|$

$\frac{1}{2}\cdot 4\cdot 2 + \frac{1}{2}\cdot 2\cdot 1$

$\quad\|$

$4+1=5.$



$$\boxed{\int_a^b f'(t)\,dt = f(b) - f(a)}$$

(d) $\int_0^4 (f'(t)+5)\,dt = \underline{\int_0^4 f'(t)\,dt} + \underline{\int_0^4 5\,dt}$

$= f(t)\Big|_0^4 + 5t\Big|_0^4$

$= \underset{0}{\underset{\|}{f(4)}} - \underset{2}{f(0)} + \underset{20}{\underset{\|}{5\cdot(4-0)}} = 18.$

22. $\tan\left(\frac{\pi}{4}+0.12\right)-\tan\left(\frac{\pi}{4}\right)=f(x)-f(a).$

linear approximation: $f(x)$

$\approx$

$$\boxed{L(x)=f(a)+f'(a)\cdot(x-a)}$$

$L(x)-f(a)=\boxed{f'(a)\cdot(x-a)}$

$\approx$

$f(x)-f(a)$

$f(x)-f(a)\sim f'(a)\cdot(x-a).$

$$\boxed{\Delta y\sim f'(a)\,\Delta x}$$

$f(x)=\tan(x),\quad a=\frac{\pi}{4},\quad \Delta x=x-a=0.12$

$f'(x)=\sec^2(x).\qquad f'(a)=\frac{1}{\cos^2\left(\frac{\pi}{4}\right)}=\frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}=2$

$\parallel$

$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}$

$\Delta y\sim 2(0.12)=0.24.$