

Definite Integral = Net Area

$$\frac{\int_a^b f(x) dx}{[a, b]} = -A_1 + A_2 - A_3 + A_4 \quad a < b$$

↑
limits of integration.

The graph shows a wavy curve labeled $y=f(x)$. Vertical dashed lines mark the points a and b on the x-axis. The regions between the curve and the x-axis are labeled A_1 , A_2 , A_3 , and A_4 . A_1 is below the x-axis, while A_2 , A_3 , and A_4 are above it. The total area is the sum of the absolute values of these areas.

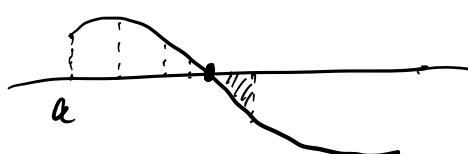
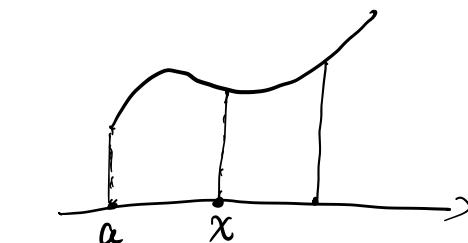
- $\int_b^a f(x) dx = -\int_a^b f(x) dx$ $\int_1^0 f(x) dx = -\int_0^1 f(x) dx$
 $= -\text{Net Area}$
 - $\int_a^a f(x) dx = -\int_a^a f(x) dx \Rightarrow \boxed{\int_a^a f(x) dx = 0}$
 - $\int_a^b f(x) dx = \int_a^b f(t) dt = \int_a^b f(u) du$
 \uparrow
 dummy variable
-
- The graph shows a function curve. A single vertical line segment connects the x-axis at point a to the curve. This represents the net area under the curve from a to a , which is zero.

Area function:

$$F(x) = \int_a^x f(t) dt$$

The graph shows a function curve $f(t)$. A vertical line is drawn from the x-axis at point a to the curve. Another vertical line is drawn from this intersection point to the point x on the x-axis. The region between the x-axis and the curve from a to x is shaded.

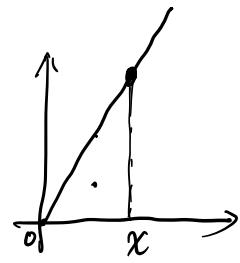
function of net area



Fundamental Theorem of Calculus (Part I).

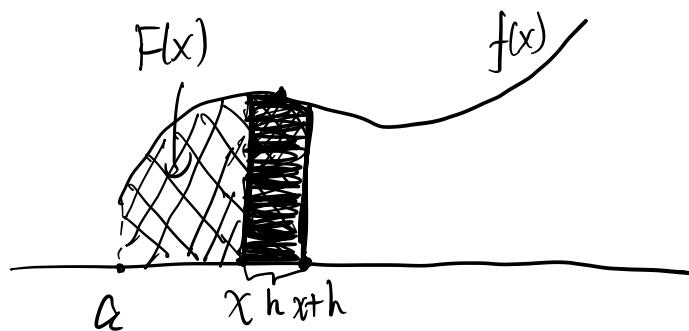
$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

Ex.: $f(x) = 2x$



$$\int_0^x f(t) dt = \frac{1}{2} \cdot x \cdot 2x = x^2$$

$$\frac{d}{dx} \int_0^x f(t) dt = \frac{d}{dx} x^2 = 2x$$



$$F(x) = \int_a^x f(t) dt$$

$$\int_a^x f(z) dz$$

$$\frac{d}{dx} F(x) = \lim_{h \rightarrow 0} \frac{F(x+h) - F(x)}{h} = f(x)$$

$$\boxed{\frac{d}{dx} \int_a^x f(t) dt = f(x).}$$

$$\frac{d}{dx} \int_a^x f(t) dt = - \frac{d}{dx} \int_a^x f(t) dt = -f(x)$$

$$f \xrightarrow{\text{integrate}} F \xrightarrow{\text{diff.}} f$$

$F \xrightarrow{\text{differentiate}} f \xrightarrow{\text{integrate}} F?$

\parallel
 F'

$$\boxed{\int_a^x F'(t) dt = F(x) + C}$$

$$\frac{d}{dx} \left[\int_a^x F'(t) dt \right] = \underline{F'(x)} = f(x) \Rightarrow C \text{ is an antiderivative of } f(x).$$

\parallel
 $G(x)$

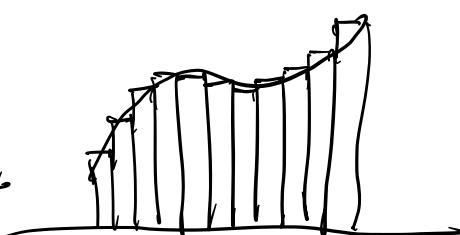
$$\underline{0} = \int_a^a F'(t) dt = \underline{F(a) + C} \Rightarrow \boxed{C = -F(a)}.$$

Then:

$$\boxed{\int_a^x F'(t) dt = F(x) - F(a)}$$

$$\boxed{\int_a^b f(t) dt = F(b) - F(a)} \text{ if } F \text{ is an antiderivative of } f$$

$$\lim_{\Delta x \rightarrow 0} \sum_{i=1}^n f(x_i) \Delta x$$



1. Find an derivative F

2. $F(b) - F(a) = \int_a^b f(t) dt.$

Ex: $\int_{-3}^4 \left(x^2 - \frac{1}{x^2} \right) dx$

1. Find antiderivative of $x^2 - x^{-2}$:

$$\frac{1}{2+1} x^{2+1} - \frac{1}{-2+1} x^{-2+1} = \frac{1}{3} x^3 + x^{-1}.$$

$$\begin{aligned} 2. \left[\frac{1}{3} x^3 + x^{-1} \right]_{-3}^4 &= \left(\frac{1}{3} 4^3 + 4^{-1} \right) - \left(\frac{1}{3} (-3)^3 + \frac{1}{-3} \right) \\ &= \frac{1}{3} (64 + 27) + \frac{1}{4} + \frac{1}{3} \\ &= \frac{91}{3} + \frac{7}{12} = \frac{364+7}{12} = \frac{371}{12} \end{aligned}$$

$$\underline{\text{Ex:}} \quad \int_{\frac{\pi}{3}}^{\pi} (2\cos(x) - \sin(x)) dx$$

$$(2\cdot \sin x - (-\cos x)) \Big|_{\frac{\pi}{3}}^{\pi}$$

$$= (2\cdot \sin \pi + \cos \pi) - (2\cdot \sin(\frac{\pi}{3}) + \cos(\frac{\pi}{3}))$$

$$= -1 - (2 \cdot \frac{\sqrt{3}}{2} + \frac{1}{2}).$$

$$= -\frac{3}{2} - \sqrt{3}.$$



Ex:

$F(x) = \int_{-1}^x t^2(7-t) dt$	$\int_{-1}^x z^2(7-z) dz$
$\cdot F(2) = \int_{-1}^2 t^2 \cdot (7-t) dt$	$\int_{-1}^x s^2(7-s) ds$
$= \int_{-1}^2 (7t^2 - t^3) dt$	
$= (7 \cdot \frac{1}{3}t^3 - \frac{1}{4}t^4) \Big _{-1}^2$	
$= (\frac{7}{3} \cdot 8 - \frac{1}{4} \cdot 16) - (\frac{7}{3} \cdot (-1) - \frac{1}{4} \cdot 1)$	$\frac{69}{4}$
$= \frac{7}{3} \cdot (8+1) - 4 + \frac{1}{4} = 21 - 4 + \frac{1}{4} = 17 + \frac{1}{4}$	$\frac{69}{4}$

- What value of $x \geq -1$ maximizes $F(x)$?

$$\frac{d}{dx} F(x) = t^2(7-t) \Big|_{t=x} = \cancel{x^2(7-x)} = 0$$

$$\Rightarrow x=0, 7.$$

$$\frac{d^2}{dx^2} F = \frac{d}{dx} x^2(7-x) = \frac{d}{dx} (7x^2 - x^3) = 14x - 3x^2 = \cancel{x(14-3x)}$$

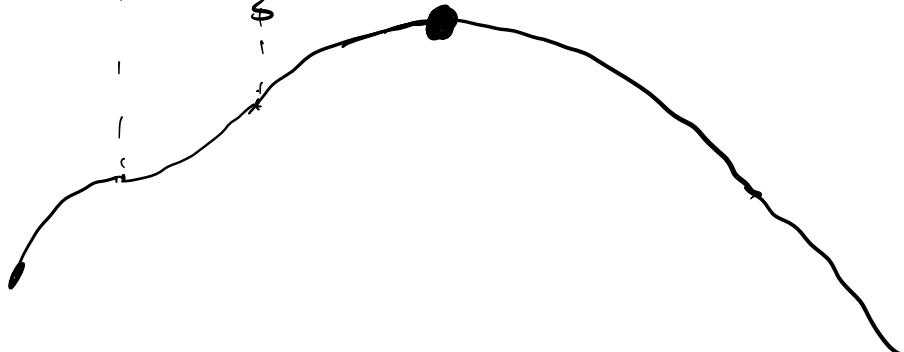
"
F"

$$F''(0)=0, \quad F''(7)=7 \cdot (14-21) < 0$$

\downarrow
0 is?

\downarrow
7 is a local maximum

$$\begin{array}{lll} F'' < 0 & F'' > 0 & F'' < 0 \\ (F' > 0) & (F' < 0) & (F' < 0) \end{array}$$



Part I: $\frac{d}{dx} \int_a^x f(t) dt = f(x).$

Part II: $\int_a^x F'(t) dt = F(x) - F(a).$

indefinite integral $\int x^p dx = \frac{1}{p+1} x^{p+1} + C, p \neq -1$

$$\int_0^x t^p dt = \frac{1}{p+1} x^{p+1} \quad p \neq -1$$

definite integral with
changing upper limit

$$\frac{d}{dx} \int_0^x t^p dt = x^p$$