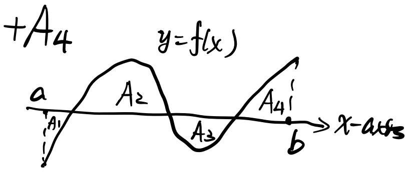
Definite Integral = Net Area

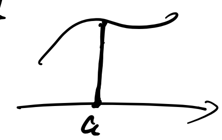
$$\int_a^b f(x)\,dx = -A_1 + A_2 - A_3 + A_4$$



[a, b] $\quad a<b$

limits of integration.

- $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ $\qquad \int_1^0 f(x)\,dx = -\int_0^1 f(x)\,dx$ $= -$ Net Area
- $\int_a^a f(x)\,dx = -\int_a^a f(x)\,dx \Rightarrow \boxed{\int_a^a f(x)\,dx = 0}$



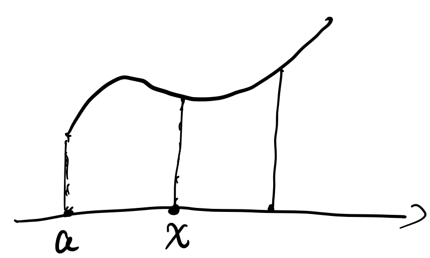
- $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(\omega)\,d\omega$

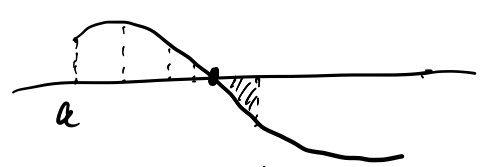
dumb variable

Area function:

$$F(x) = \int_a^{x} f(t)\,dt$$

function of net area

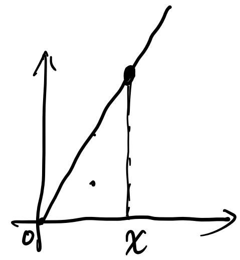




Fundamental Theorem of Calculus (Part I).

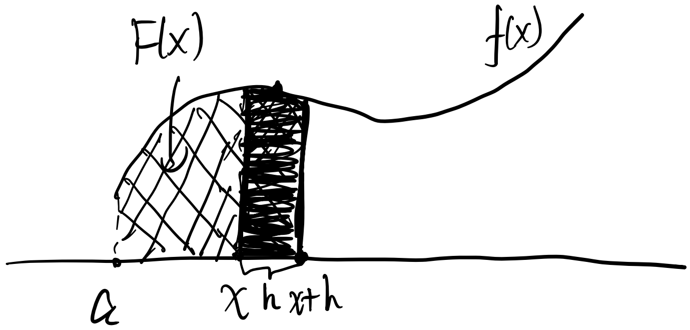
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Ex: $f(x) = 2x$



$$\int_0^x f(t)\,dt = \frac{1}{2}\cdot x \cdot 2x = x^2$$

$$\frac{d}{dx}\int_0^x f(t)\,dt = \frac{d}{dx}x^2 = 2x$$



$$F(x) = \int_a^x f(t)\,dt = \int_a^x f(z)\,dz$$

$$\frac{d}{dx}F(x) = \lim_{h\to 0}\frac{F(x+h)-F(x)}{h} = f(x)$$

$$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x).}$$

$$\frac{d}{dx}\int_x^a f(t)\,dt = -\frac{d}{dx}\int_a^x f(t)\,dt = -f(x)$$

$$f \xrightarrow{\text{integrate}} F \xrightarrow{\text{diff.}} f$$

$$F \xrightarrow{\text{differentiate}} \underset{\underset{F'}{\parallel}}{f} \xrightarrow{\text{integrate}} F?$$

$$\boxed{\int_a^x F'(t)\,dt = F(x) + C}$$

$$\underbrace{\frac{d}{dx}\int_a^x F'(t)\,dt}_{G(x)} = F'(x) = f(x) \Rightarrow$$ G is an antiderivative of $f(x)$.

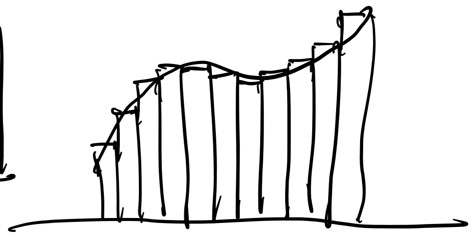
$$0 = \int_a^a F'(t)\,dt = F(a) + C \Rightarrow \boxed{C = -F(a)}.$$

Thm: $$\boxed{\int_a^x F'(t)\,dt = F(x) - F(a)}$$

$$\boxed{\int_a^b f(t)\,dt = F(b) - F(a) \quad \text{if } F \text{ is an antiderivative of } f}$$

$$\parallel$$

$$\boxed{\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\,\Delta x}$$



1. Find an derivative F

2. $F(b)-F(a) = \int_a^b f(t)\,dt$.

Ex: $\int_{-3}^{4} \left(x^2 - \frac{1}{x^2} \right) dx$

1. Find antiderivative of $x^2 - x^{-2}$:

$\frac{1}{2+1} x^{2+1} - \frac{1}{-2+1} x^{-2+1} = \frac{1}{3} x^3 + x^{-1}$.

2. $\left(\frac{1}{3} x^3 + x^{-1} \right) \Big|_{-3}^{4} = \left(\frac{1}{3} 4^3 + 4^{-1} \right) - \left(\frac{1}{3} (-3)^3 + \frac{1}{-3} \right)$

$= \frac{1}{3} (64 + 27) + \frac{1}{4} + \frac{1}{3}$

$= \frac{91}{3} + \frac{7}{12} = \frac{364+7}{12} = \frac{371}{12}$

Ex: $\int_{\frac{\pi}{3}}^{\pi} (2\cos(x) - \sin(x))\,dx$

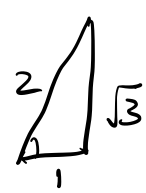
$\parallel$

$(2\cdot\sin x - (-\cos x))\Big|_{\frac{\pi}{3}}^{\pi}$

$= (2\cdot\sin\pi + \cos\pi) - (2\cdot\sin(\frac{\pi}{3}) + \cos(\frac{\pi}{3}))$

$= -1 - (2\cdot\frac{\sqrt{3}}{2} + \frac{1}{2})$

$= -\frac{3}{2} - \sqrt{3}$.



Ex: $\boxed{F(x) = \int_{-1}^{x} t^2(7-t)\,dt} = \int_{-1}^{x} z^2(7-z)\,dz$

$\parallel$

$\int_{-1}^{x} s^2(7-s)\,ds$

• $F(2) = \int_{-1}^{2} t^2\cdot(7-t)\,dt$

$= \int_{-1}^{2} (7t^2 - t^3)\,dt$

$= (7\cdot\frac{1}{3}t^3 - \frac{1}{4}t^4)\Big|_{-1}^{2}$

$= (\frac{7}{3}\cdot 8 - \frac{1}{4}\cdot 16) - (\frac{7}{3}\cdot(-1) - \frac{1}{4}\cdot 1)$ $\quad \frac{69}{4}$

$\parallel$

$= \frac{7}{3}\cdot(8+1) - 4 + \frac{1}{4} = 21 - 4 + \frac{1}{4} = 17 + \frac{1}{4}$

- What value of $x \geq -1$ maximizes $F(x)$?

$$\frac{d}{dx}F(x) = t^2(7-t)\Big|_{t=x} = x^2(7-x) = 0$$

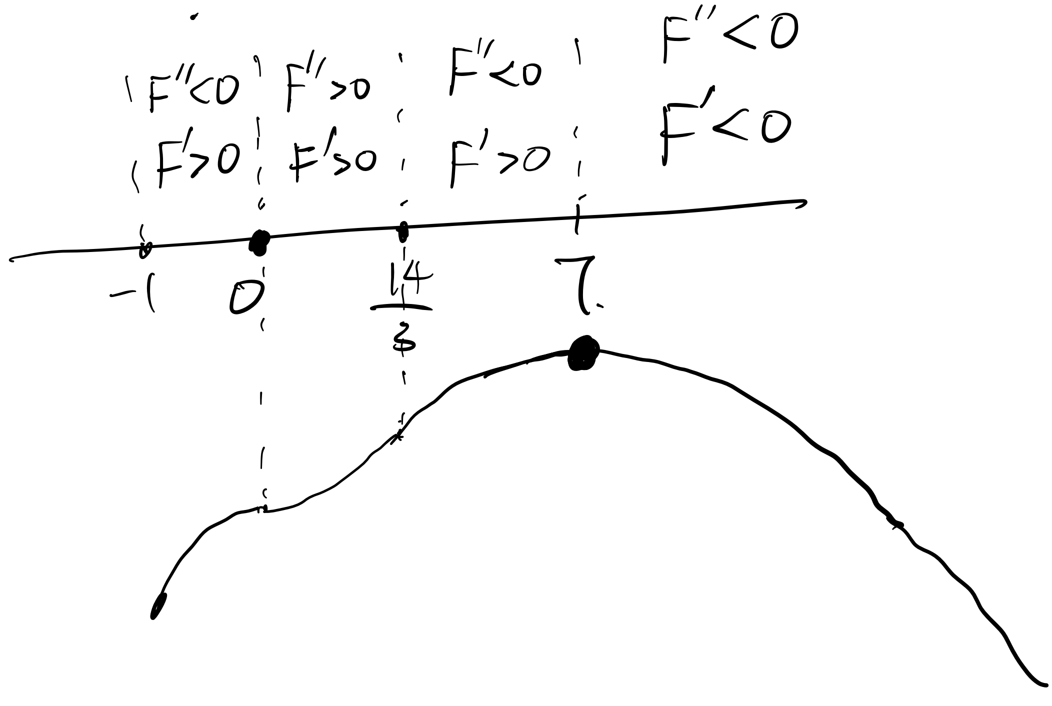
$\Rightarrow x = 0, 7.$

$$\underset{\overset{\|}{F''}}{\frac{d^2}{dx^2}} F = \frac{d}{dx}x^2(7-x) = \frac{d}{dx}(7x^2 - x^3) = 14x - 3x^2 = x(14-3x).$$

$F''(0) = 0$. $\quad F''(7) = 7\cdot(14-21) < 0$

$\Downarrow \qquad\qquad\qquad \Downarrow$

0 is ? $\qquad$ 7 is a local maximum



Part I: $\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$.

Part II: $\int_a^x \underbrace{F'(t)}_{= f(t)}\,dt = F(x) - F(a)$.

indefinite integral $\rightarrow$ $\boxed{\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C,\quad p \neq -1}$

$\boxed{\int_0^x t^p\,dt = \frac{1}{p+1}x^{p+1} \qquad p \neq -1}$

definite integral with changing upper limit

$\boxed{\frac{d}{dx}\int_0^x t^p\,dt = x^p}$