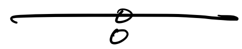
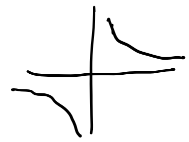
Ex: $f(x) = 3x^2 + \frac{6}{x}$

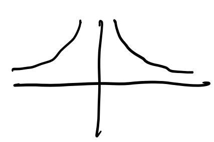


1. Identity the domain: $(-\infty, 0) \cup (0, +\infty)$



2. Exploit symmetry: odd: $f(-x) = -f(x) \rightarrow$

even: $f(-x) = f(x) \rightarrow$



f is not odd or even

3. Find 1st and 2nd derivatives.

$f = 3x^2 + 6x^{-1}$: $f'(x) = 6x - 6x^{-2}$, $f'' = 6 - 6(-2)x^{-3} = 6 + 12x^{-3}$

4. Find critical points and possible inflection points.

$$f'(x) = 0 = 6x - 6x^{-2} = 6x^{-2} \cdot (x^3 - 1) = 0 \Leftrightarrow x^3 = 1 \Leftrightarrow x = 1$$

critical point.

$$= \frac{6}{x^2}(x^3 - 1)$$

$$f''(x) = 0 = 6 + 12 \cdot x^{-3} = 0 \Leftrightarrow \frac{12}{x^3} = -6 \Leftrightarrow x^3 = -2 \Leftrightarrow x = (-2)^{\frac{1}{3}}$$

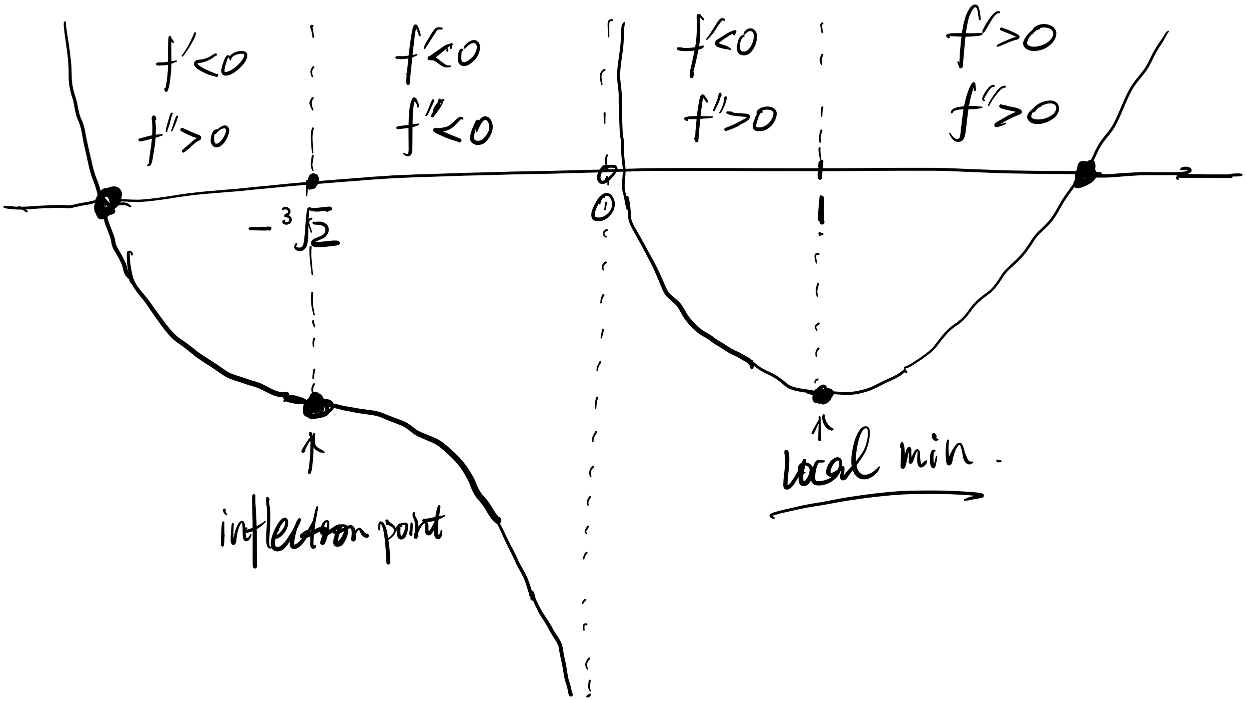
$$-\sqrt[3]{2} = -2^{\frac{1}{3}}$$

possible inflection point

$$6 + \frac{12}{x^3} = \frac{1}{x^3} 6 \cdot \underbrace{(x^3 + 2)}_{x^3 - (-2)}$$

5. Find intervals on which the function is increasing / decreasing

concave up / down.



6. Identify extremal values and inflection points.

$$f(1) = 3x^2 + \frac{6}{x}\Big|_{x=1} = 3+6=9$$

7. Locate all asymptotes and determine end behavior

$f(x)=3x^2+\frac{6}{x}$, $\lim_{x\to 0^-} f(x) = 0+\frac{6}{0^-} = -\infty$, $\lim_{x\to 0^+} 0+\frac{6}{0^+} = +\infty$

$$\lim_{x\to+\infty} f(x) = +\infty, \quad \lim_{x\to-\infty} f(x) = +\infty$$

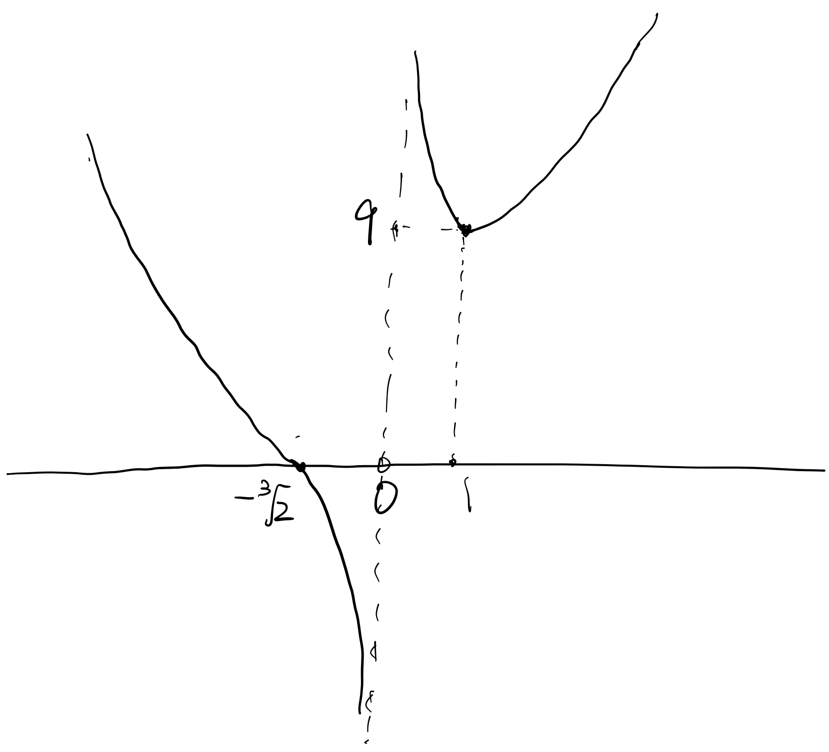
$$\underline{f(x) = 3x^2 + \frac{6}{x}} \underset{x\to\pm\infty}{\sim} \underline{3x^2}$$

8. Find the intercepts : intersection pts. of the graph with y-axis and x-axis.

Set $y = f(x) = 0 = 3x^2 + \frac{6}{x} \Leftrightarrow 3x^2 = -\frac{6}{x}$

$\Leftrightarrow x^3 = -2$

$\Rightarrow \boxed{x = (-2)^{\frac{1}{3}} = -2^{\frac{1}{3}}}$



Ex: $\underline{f'(x) = x(x-9)^2}$

$$f''(x) = 1\cdot(x-9)^2 + x\cdot 2\cdot(x-9)\cdot 1$$

$$= (x-9)\cdot(x-9+2x)$$

$$= (x-9)\cdot 3\cdot(x-3)$$

- critical points: $\underline{f'(x)=0 = x(x-9)^2}$

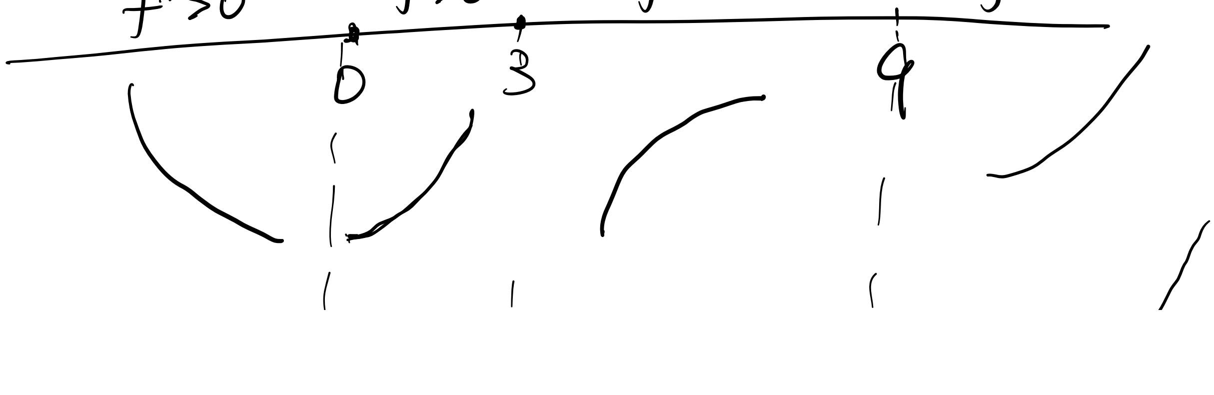
$$\Rightarrow \quad x=0,\ 9$$

possible inflection points: $\underline{f''(x)=0 = (x-3)\cdot(x-9)\cdot 3}$

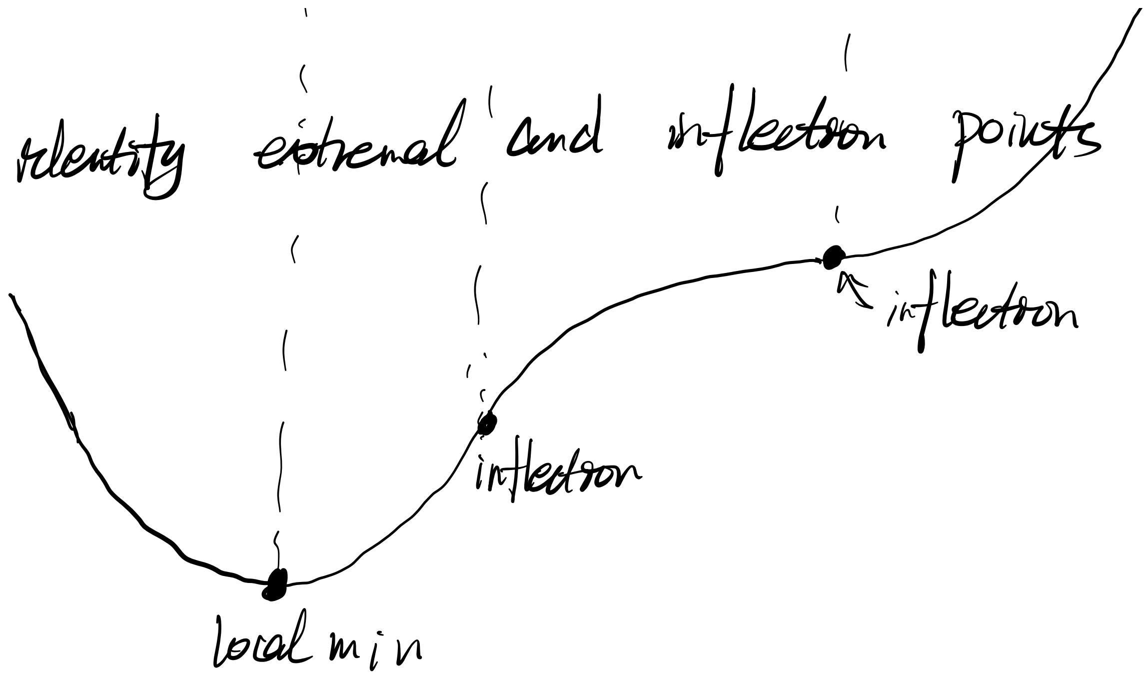
$$\Rightarrow \quad x = 3,\ 9$$

- intervals of increasing/decreasing, concave up/down

$f'<0$	$f'>0$	$f'>0$	$f'>0$
$f''>0$	$f''>0$	$f''<0$	$f''>0$



- identify extremal and inflection points

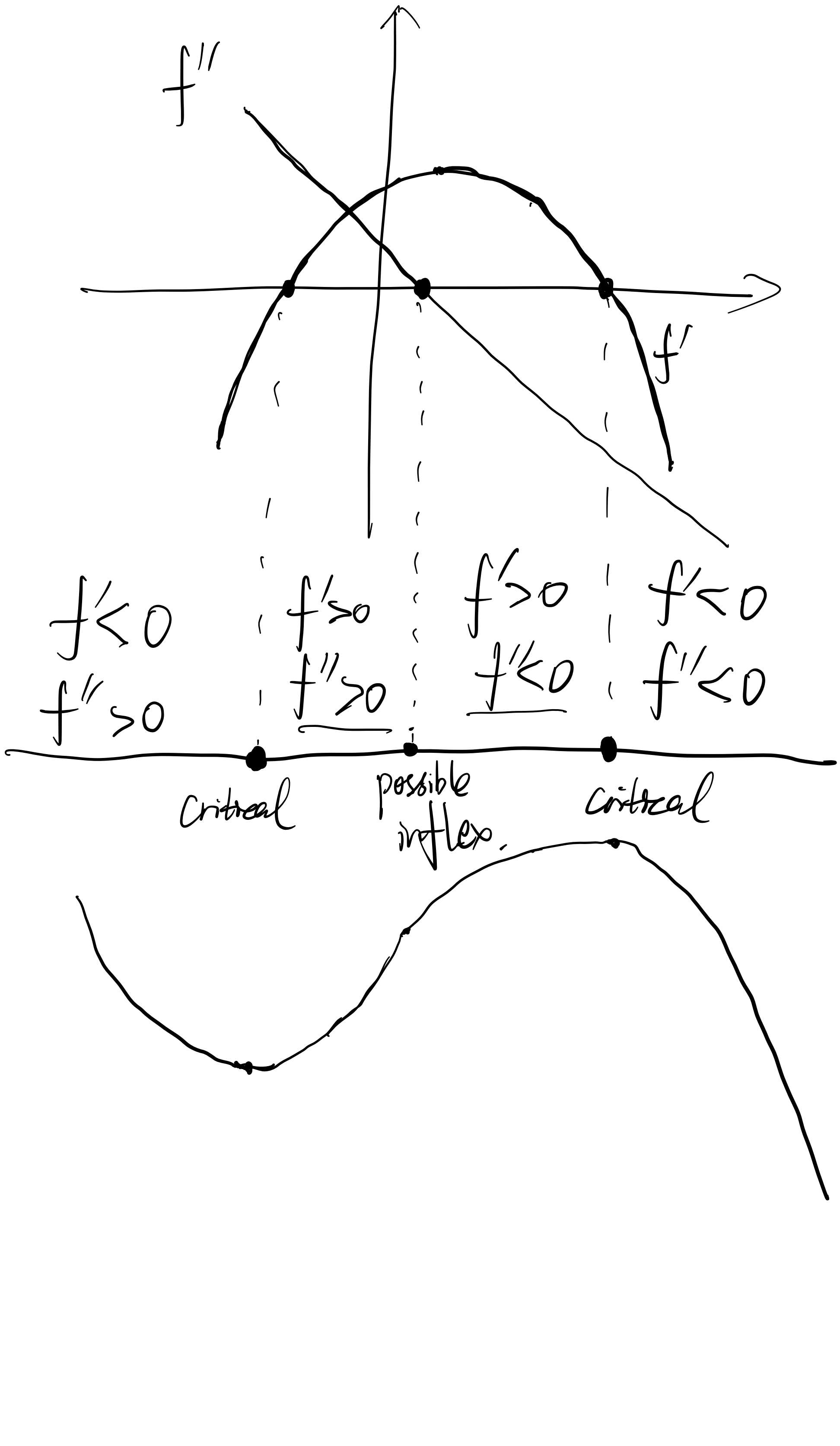


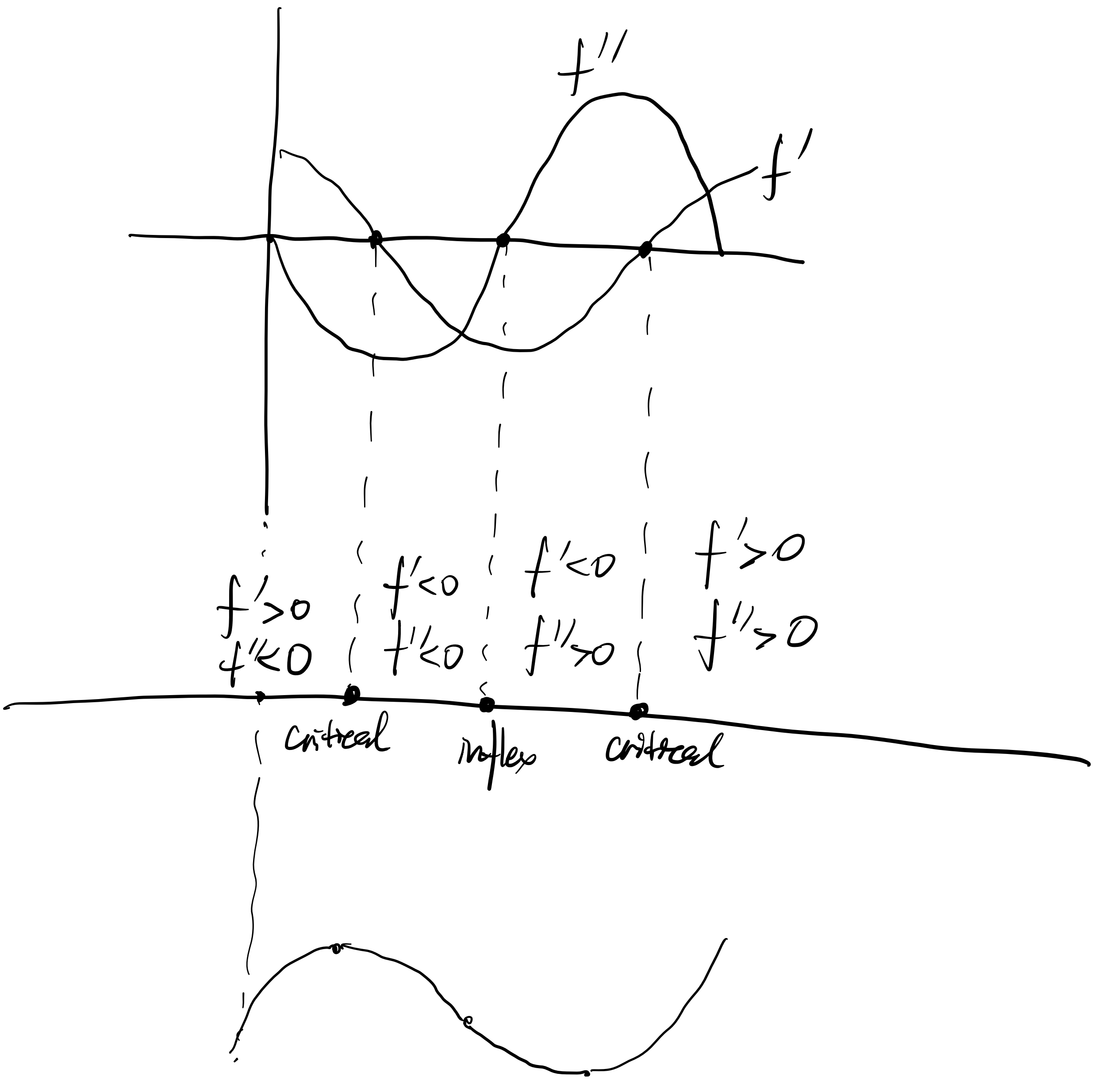
$$f'(x) = x(x-9)^2 = x\cdot(x^2-18x+9)$$

$$= x^3 - 18x^2 + 9x$$

$$\Rightarrow \quad f(x) = \frac{x^4}{4} - 18\cdot\frac{x^3}{3} + 9\cdot\frac{x^2}{2} + C$$

Ex:





x^3 odd

x^2 even