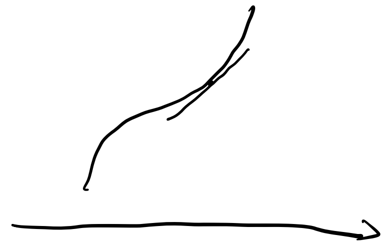
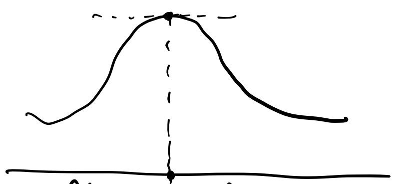
$f'(x) > 0 \iff$ increasing

$f'(x) < 0 \iff$ decreasing

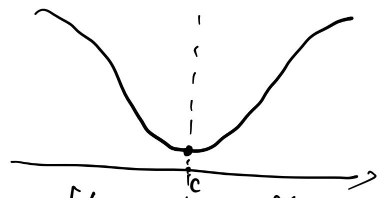




$f' > 0$ $f' < 0$

$f'(c) = 0$

critical point



c

$f' < 0$ $f' > 0$

$f'(c) = 0$

critical pt.

First derivative Test: $f': + \xrightarrow{c} -$ local maximum

$- \xrightarrow{c} +$ local minimum.

Ex: $\boxed{f(x) = 2x^3 + 6x^2 - 48x + 8}$ on $[-4, 6]$.

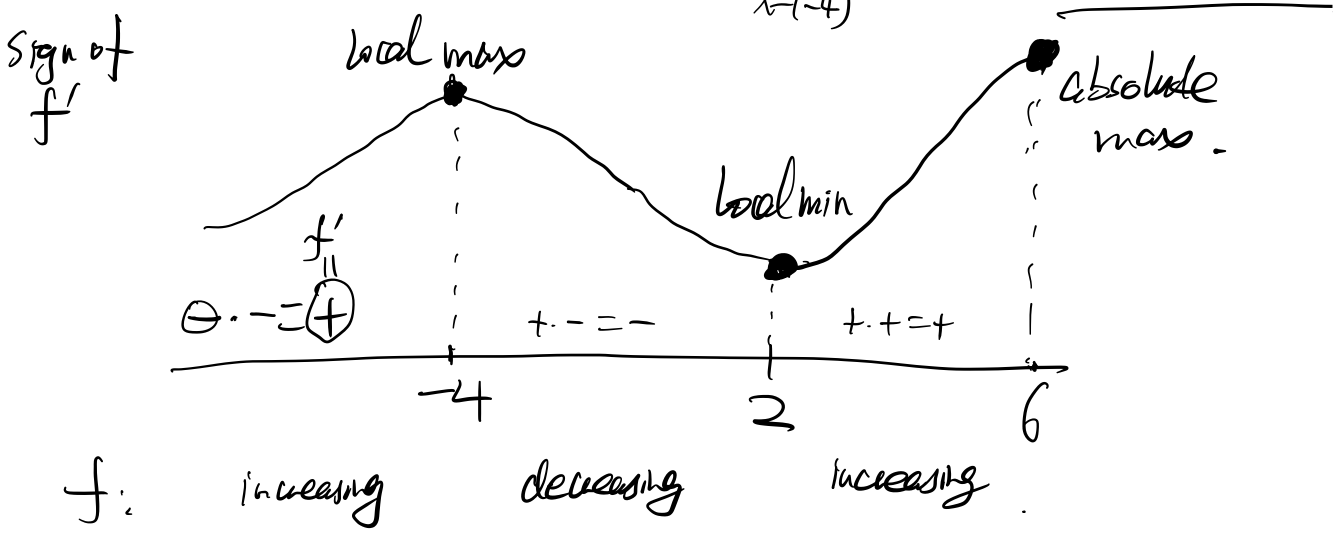
Find local and absolute extremal values of f.

Sol: • Find critical points.

$$f'(x) = 6x^2 + 12x - 48 = 0$$
$$= 6(x^2 + 2x - 8)$$
$$= 6 \cdot (x+4)(x-2)$$

$\Rightarrow$ critical points: $x = -4$ and 2.

• at $x=-4$, $\boxed{f'(x)=6\underbrace{(x+4)}_{x-(-4)}(x-2)}$ local maximum



$x=2$ local minimum.

• $f(-4) = 2x^3+6x^2-48x+8 \Big|_{x=-4}$

$= 2\cdot(-4)^3+6\cdot(-4)^2-48\cdot(-4)+8$

$= 2\cdot(-64)+96+192+8$

$= -128+296 = 168$

$f(2) = 2\cdot 2^3+6\cdot 2^2-48\cdot 2+8 = 16+24-56+8$

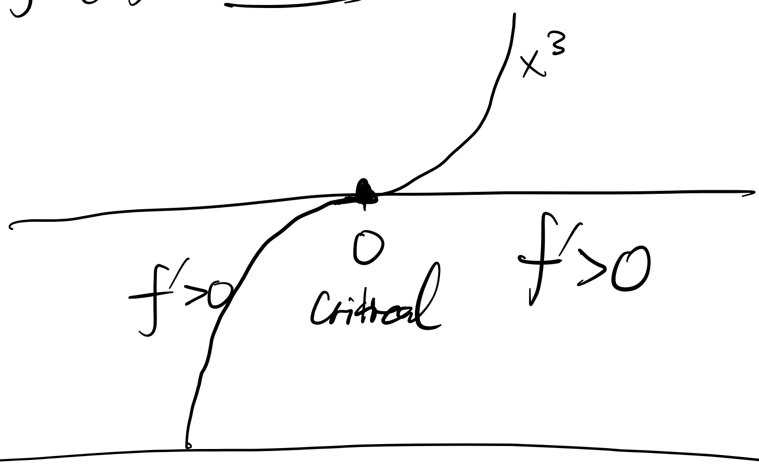
$= 48-56 = -8$. absolute minimum

$f(6) = 2\cdot 6^3+6\cdot 6^2-48\cdot 6+8$

$= 18\cdot 6^2-8\cdot 6^2+8 = 10\cdot 36+8 = \boxed{368}$ absolute maximum

$f(x) = x^3$

critical points $\quad f'(x) = \underline{3x^2} = 0 \Rightarrow x = 0$

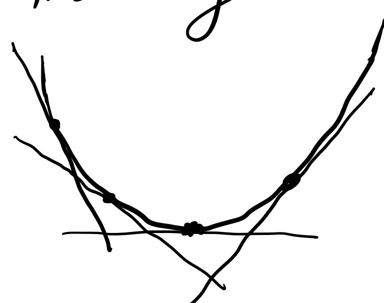


Sign of $f''(x) = \frac{d}{dx} f'$

$f'' > 0 \Rightarrow f'$ is increasing $\Rightarrow$ f is concave up

$\parallel$

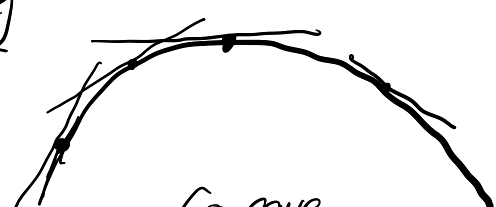
$\frac{d}{dx} f'$



$\underline{f'' < 0} \Rightarrow \underline{f' \text{ is decreasing}}$

$\Downarrow$

f is concave down



Ex: $f(x) = 2x^3 + 6x^2 - 48x + 8$

$$f'(x) = \underline{6x^2 + 12x - 48}$$
$$= 6\cdot(x+4)(x-2) \Rightarrow \text{critical points } \underline{-4,\ 2}$$

$$f''(x) = \frac{d}{dx} f' = 12x + 12 = \underline{12\cdot(x+1)}$$

$$\text{Sign of } f''(x) = \begin{cases} - & x < -1 & \frown \\ 0 & x = -1 & \\ + & x > -1 & \smile \end{cases}$$

At $x = -1$: f changes from concave down to concave up.

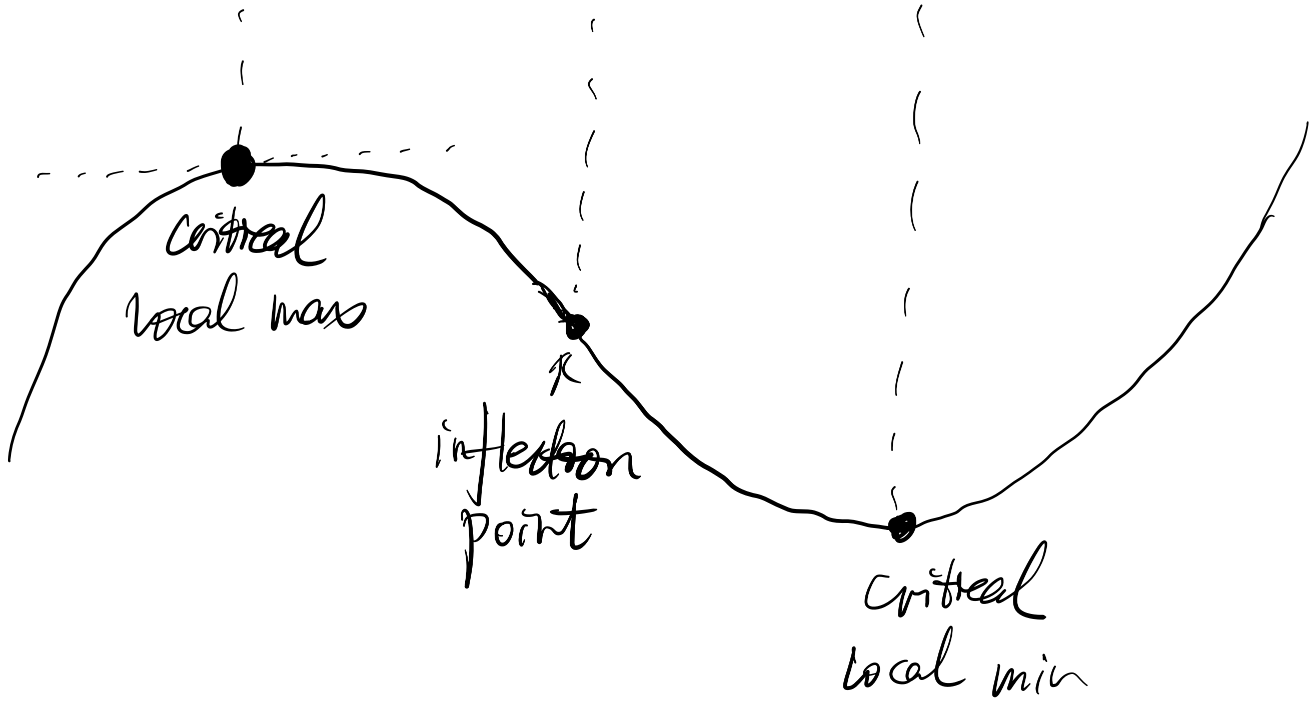
$\Rightarrow$ $\underline{x = -1}$ is an $\underline{\text{inflection point.}}$

the concavity changes at this point

$f' = 0$

$f'' < 0$

$f' > 0$		$f' < 0$		$f' < 0$		$f' > 0$
$f'' < 0$		$f'' < 0$		$f'' > 0$		$f'' > 0$
increasing	-4	decreasing	-1	decreasing	2	increasing
concave down		concave down		concave up		concave up



2nd derivative test for Local Extrema:

Suppose f is very smooth and $f'(c)=0$.

- If $\boxed{f''(c)>0}$, then $f' : - \to +$

  $\Rightarrow$ c is local min.

- If $f''(c)<0$, then $f' : + \to -$

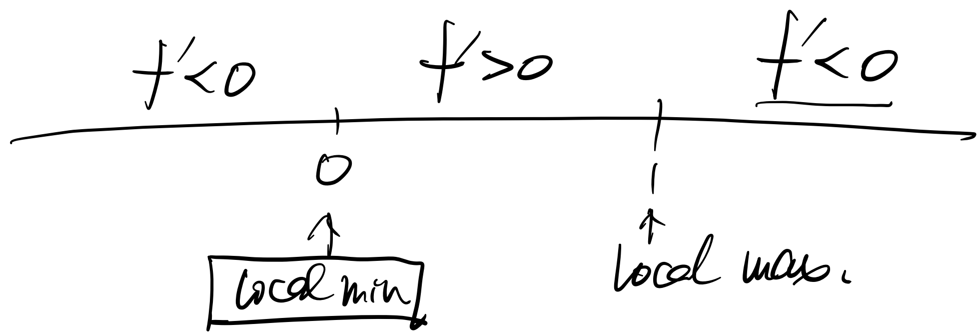
  $\Rightarrow$ c is local max

- If $f''(c)=0$, then inconclusive.

Ex: $f(x) = 3x^2 \boxed{e^{-2x}} + 5$

- Find critical points.

$$f'(x) = 6x\cdot e^{-2x} + 3x^2\cdot e^{-2x}\cdot(-2) + 0$$
$$= 6x\cdot e^{-2x} - 6x^2\cdot e^{-2x}$$
$$= 6x\cdot(1-x)\cdot e^{-2x} = 0$$
$$\Rightarrow x = 0, 1$$



$$f'(x) = 6(x-x^2)\cdot e^{-2x}$$

- $f''(x) = 6\cdot(1-2x)\cdot e^{-2x} + 6(x-x^2)e^{-2x}\cdot(-2)$

$$= 6e^{-2x}(1-2x-2(x-x^2))$$

$$= 6\cdot e^{-2x}\cdot\boxed{(2x^2-4x+1)}$$

$f''(0) = 6\cdot 1\cdot 1 = 6 > 0$ local min

$f''(1) = 6\cdot e^{-2}\cdot(\underbrace{2-4+1}_{-1}) = -6e^{-2} < 0$

$\Rightarrow$ local max

- inflection point $f''(x)=0$:

$$2x^2-4x+1=0 \Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$1 \pm 0.7 = \boxed{1 \pm \frac{\sqrt{2}}{2}} = \frac{4 \pm \sqrt{16-8}}{4}$$

$f'<0$	$f'>0$	$f'>0$	$f'<0$	$f'<0$
$f''>0$	$f''>0$	$f''<0$	$f''<0$	$f''>0$

$0 \qquad 1-\frac{\sqrt{2}}{2} \qquad 1 \qquad 1+\frac{\sqrt{2}}{2}$