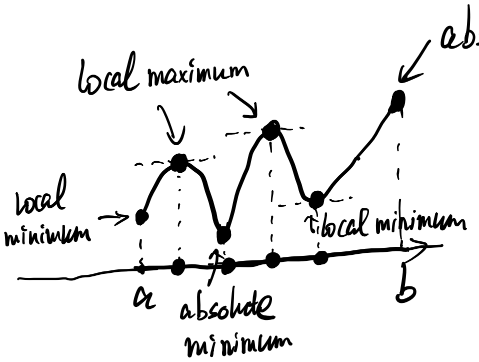
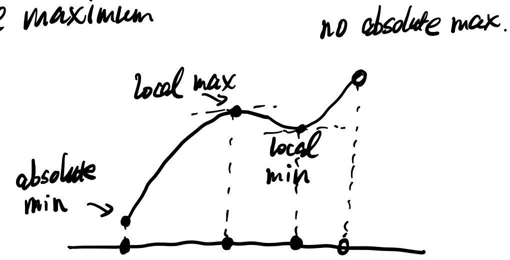
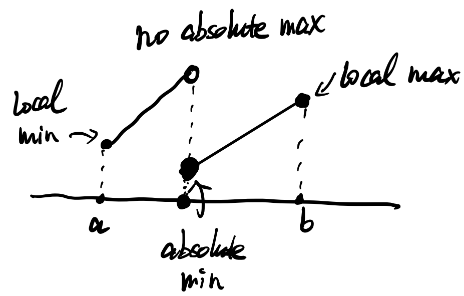






- Locate critical points

Critical points: $c \in (a, b)$ is called a critical point if $f'(c) = 0$ or $f'(c)$ does not exist.

- Calculate the values of f at critical points.
- If f is defined at a or b, then calculate $f(a), f(b)$
- Compare to find the absolute maximum and minimum

Ex: $f(x) = \dfrac{2x}{x^2+4}$ $\quad x \in (-\infty, +\infty)$

- Find critical points:

$$f'(x) = \frac{2\cdot(x^2+4) - \underset{\substack{\| \\ 4x^2}}{2x\cdot 2x}}{(x^2+4)^2} = \frac{8-2x^2}{(x^2+4)^2} = 0$$

$$\Leftrightarrow 8-2x^2=0 \Leftrightarrow x^2=4 \Leftrightarrow x = \pm 2.$$

- $f(2) = \dfrac{2\cdot 2}{2^2+4} = \dfrac{4}{8} = \dfrac{1}{2}.$

$f(-2) = \dfrac{2\cdot(-2)}{(-2)^2+4} = \dfrac{-4}{8} = -\dfrac{1}{2}$

- $f(-\infty) = \lim\limits_{x\to-\infty} \dfrac{2x}{x^2+4} = \lim\limits_{x\to-\infty} \dfrac{\frac{2}{x}}{1+\frac{4}{x^2}} = 0.$

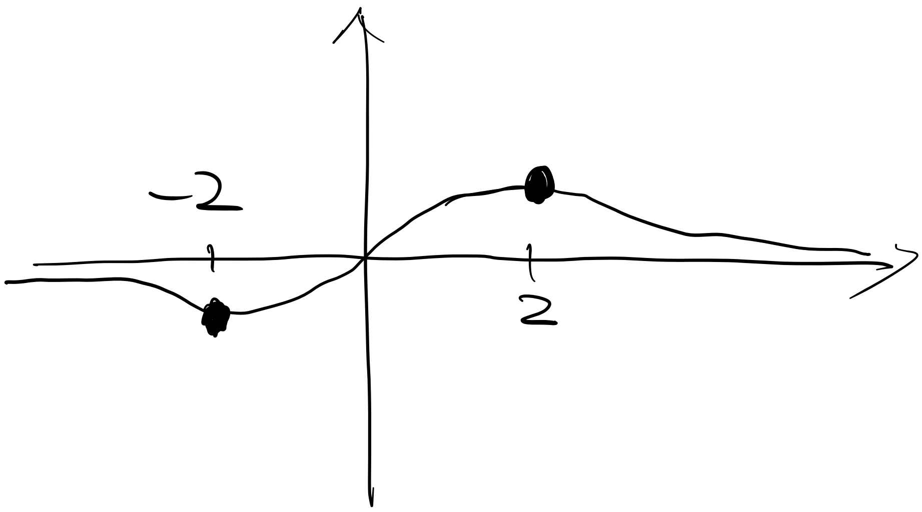
$f(+\infty) = 0$

- absolute minimum is $f(-2) = -\dfrac{1}{2}$

maximum is $f(2) = \dfrac{1}{2}$

- $f'(x) = \dfrac{2(4-x^2)}{(x^2+4)^2}$

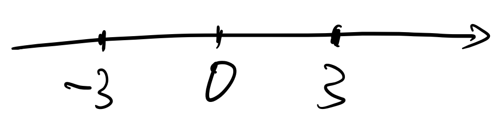
$$云 \begin{cases} >0 & |x|<2 \\ =0 & |x|=2 \\ <0 & |x|>2 \end{cases}$$



Ex: $f(x) = x \cdot \sqrt{9-x^2}$

$$9-x^2 \geq 0$$
$$\Updownarrow$$
$$x^2 \leq 9$$
$$\Downarrow$$
$$|x| = \sqrt{x^2} \leq \sqrt{9} = 3$$

domain: $[-3, 3]$



- Find critical points.

$(f \cdot g)' = f' \cdot g + f \cdot g'$

$$f' = \sqrt{9-x^2} + x \cdot \frac{1}{2}(9-x^2)^{-\frac{1}{2}} \cdot \frac{d}{dx}(9-x^2) = -2x$$

$$= \sqrt{9-x^2} - \frac{x^2}{\sqrt{9-x^2}}$$

$$= \frac{1}{\sqrt{9-x^2}}(9-x^2-x^2) = \frac{1}{\sqrt{9-x^2}}(9-2x^2)$$

$$f' = 0 \iff 9-2x^2=0 \iff x^2 = \frac{9}{2}$$

$$\iff x = \frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}$$

- $f\left(\frac{3}{\sqrt{2}}\right) = x \cdot \sqrt{9-x^2}\Big|_{x=\frac{3}{\sqrt{2}}}$

$$= \frac{3}{\sqrt{2}} \cdot \sqrt{9-\frac{9}{2}} = \frac{3}{\sqrt{2}} \cdot \frac{3}{\sqrt{2}} = \frac{9}{2}$$

$$f\left(-\frac{3}{\sqrt{2}}\right) = -\frac{9}{2}$$

- $f(3) = 3 \cdot \sqrt{9-3^2} = 0$

  $f(-3) = -3 \cdot \sqrt{9-(-3)^2} = 0$.

- $f\left(\frac{3}{\sqrt{2}}\right) = \frac{9}{2}$ absolute maximum

  $f\left(-\frac{3}{\sqrt{2}}\right) = -\frac{9}{2}$ ...... minimum

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Ex: $f(x) = x^{\frac{1}{3}} - x^2$ on $[-8, 8]$

- critical points:

  $f'(x) = \frac{1}{3} \cdot x^{-\frac{2}{3}} - 2 \cdot x = 0$

  $\frac{1}{3} \cdot \frac{1}{x^{\frac{2}{3}}} = 2x \Rightarrow 2 \cdot x \cdot 3 \cdot x^{\frac{2}{3}} = 1$

  $\shortparallel$

  $6 \cdot x^{\frac{5}{3}} = 1$

  $\Downarrow$

  $\boxed{x = \left(\frac{1}{6}\right)^{\frac{3}{5}}} \Leftarrow x^{\frac{5}{3}} = \frac{1}{6}$

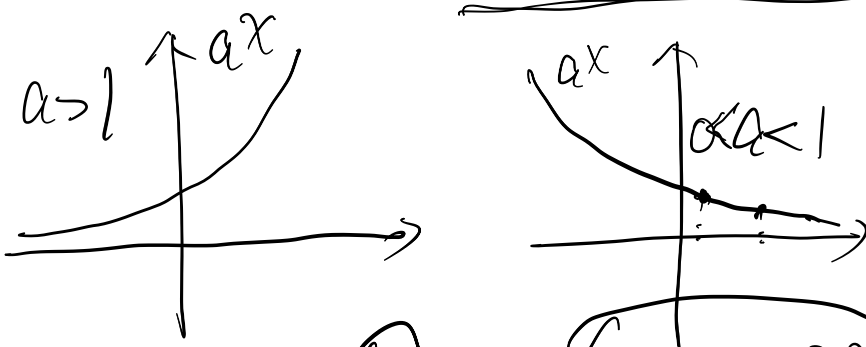
critical points are:

$\frac{1}{6^{\frac{3}{5}}}$ and 0

$(32)^{\frac{3}{5}} = \left((32)^{\frac{1}{5}}\right)^{3} = 2^{3} = 8$

$$f\left(\left(\frac{1}{6}\right)^{\frac{3}{5}}\right) = \left(\frac{1}{6}\right)^{\frac{3}{5}\cdot\frac{1}{3}} - \left(\frac{1}{6}\right)^{\frac{3}{5}\cdot 2}$$

$$= \left(\frac{1}{6}\right)^{\frac{1}{5}} - \left(\frac{1}{6}\right)^{\frac{6}{5}} > 0$$



$$f(0) = 0 < f\left(\left(\frac{1}{6}\right)^{\frac{3}{5}}\right)$$

$$f(-8) = (-8)^{\frac{1}{3}} - (-8)^{2} = -2 - 64 = -66$$

$$f(8) = 8^{\frac{1}{3}} - 8^{2} = 2 - 64 = -62$$

- absolute maximum $f\left(\left(\frac{1}{6}\right)^{\frac{3}{5}}\right) > 0$

$$\| $$

$$\left(\frac{1}{6}\right)^{\frac{1}{5}} - \left(\frac{1}{6}\right)^{\frac{6}{5}}$$

- absolute minimum: $f(-8) = -66$

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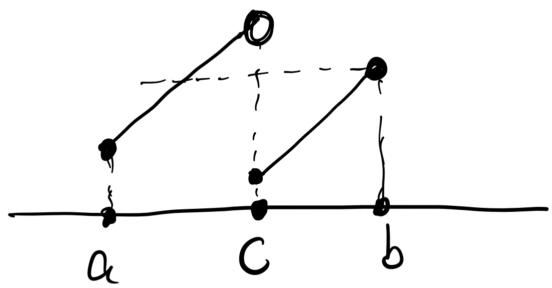
no absolute maximum



Fact: Let f be a function defined on a closed interval. If f is continuous, then there exist absolute maximum and absolute minimum.

$f(c)$

$\wedge$

$f(a) < f(b)$

absolute max.