

$$\boxed{\frac{df}{dt} = 4} \text{ cm/min}$$

$$\boxed{\frac{dx}{dt} = ?}$$

$$f^2 = x^2 + y^2$$

$$f^2 = x^2 + (x^2)^2$$

$$\boxed{f^2 = x^2 + x^4}$$

$$2f \cdot \frac{df}{dt} = 2x \cdot \frac{dx}{dt} + 4x^3 \cdot \frac{dx}{dt}$$

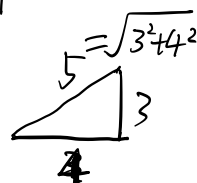
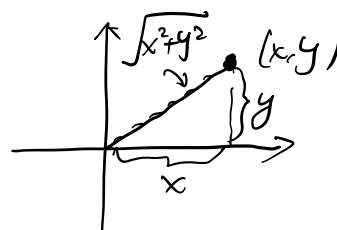
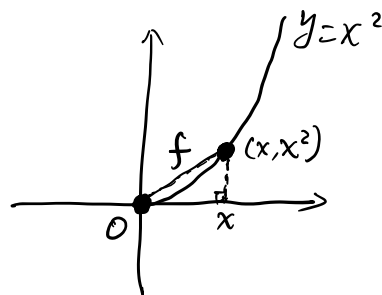
$$= \left(\frac{dx}{dt} \right) (2x + 4x^3)$$

$$\frac{dx}{dt} = \frac{2f}{2x + 4x^3} \cdot \frac{df}{dt} = \frac{f}{x + 2x^3} \cdot \left(\frac{df}{dt} \right)$$

at point (4, 16), $x = 4$, $f = \sqrt{4^2 + 16^2} = 4 \cdot \sqrt{1 + 4^2} = 4 \cdot \sqrt{17}$

$\frac{4^2 + 4^2 \cdot 4^2}{4^2 + 4^2 \cdot 4^2}$

$$\frac{dx}{dt} = \frac{4\sqrt{17}}{4 + 2 \cdot 4^3} \cdot 4 = \frac{4\sqrt{17}}{1 + 2 \cdot 4^2} = \frac{4\sqrt{17}}{33}$$



b) $y = x^2$

$$\frac{dy}{dt} = 2x \frac{dx}{dt}$$

$$\frac{dy}{dx} = 2x$$

$$= \frac{dy}{dx} \frac{dx}{dt}$$

c) $\frac{dy}{dt} = 2x \cdot \frac{dx}{dt} = 2 \cdot 4 \cdot \frac{4\sqrt{17}}{33} = \frac{32\sqrt{17}}{33}$

$$D = \sqrt{x^2 + x^4} \quad \frac{dD}{dt} = \frac{1}{2}(x^2 + x^4)^{-\frac{1}{2}} \left(2x \frac{dx}{dt} + 4x^3 \cdot \frac{dx}{dt} \right)$$

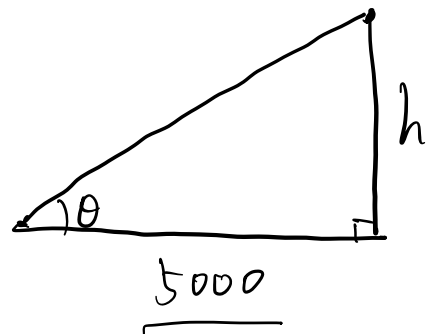
$$= (x^2 + x^4)^{-\frac{1}{2}} (x + 2x^3) \cdot \frac{dx}{dt}$$

$$= \frac{x + 2x^3}{\sqrt{x^2 + x^4}} \frac{dx}{dt}$$

$$\frac{x + 2x^3}{\sqrt{x^2(1+x^2)}}$$

$$\frac{1 + 2x^2}{\sqrt{1+x^2}}$$

$$\boxed{\frac{dh}{dt} = 600} \text{ ft/s.}$$



$$\frac{d}{dt}(\cos \theta) = -\sin \theta \cdot \frac{d\theta}{dt} < 0$$

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$$\frac{d^2h}{dt^2} = \frac{d}{dt} \left(\frac{dh}{dt} \right) \text{ velocity}$$

= acceleration

not enough
information

$$\frac{d\theta}{dt} = ?$$

$$\tan \theta = \frac{h}{5000} = \frac{1}{5000} \cdot h$$

$$\sec^2 \theta \cdot \frac{d\theta}{dt} = \frac{1}{5000} \cdot \frac{dh}{dt}$$

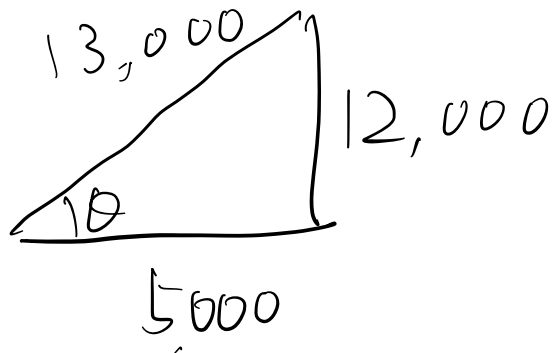
$$\frac{d\theta}{dt} = \frac{1}{5000 \sec^2 \theta} \cdot \frac{dh}{dt}$$

$$\frac{d\theta}{dt} = \frac{\left(\frac{5}{13}\right)^2}{5000} \cdot 600$$

$$\frac{\cos^2 \theta}{5000} \frac{dh}{dt}$$

$$\frac{1}{169} \cdot \frac{5^2 \times 6}{5 \times 10}$$

$$5^2 + 12^2 = 13^2$$



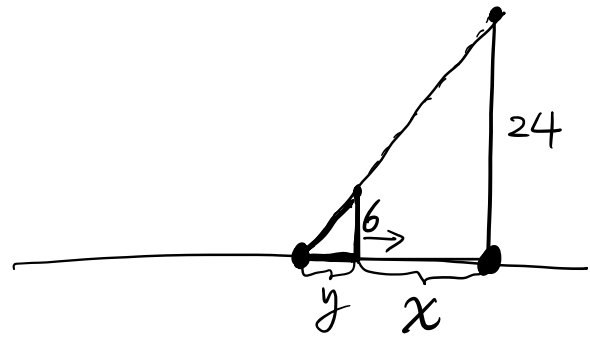
$$\cos \theta = \frac{5}{13}$$

$$\frac{d\theta}{dt} = \frac{5 \times 6}{169 \times 10} = \frac{5 \times 6}{1690} = \frac{3}{169}$$

$$\underline{\frac{dx}{dt} = -6}$$

$$\boxed{\frac{dy}{dt} = ?}$$

$$\underline{\frac{d}{dt}(x+y) = \frac{dx}{dt} + \frac{dy}{dt}}$$



$$\boxed{\frac{1}{4} = \frac{6}{24} = \frac{y}{y+x}} \quad y = \frac{1}{3}x$$

$$\boxed{y+x = 4 \cdot y} \Leftrightarrow \underline{x = 3y}$$

$$\underline{\frac{dy}{dt} = \frac{1}{3} \cdot \frac{dx}{dt} = \frac{1}{3} \cdot (-6) = -2}$$

$$\frac{dx}{dt} = 3 \frac{dy}{dt}$$

$$\frac{d}{dt}(x+y) = -6 + (-2) = -8$$